Optimal Capital Requirement with Noisy Signals on Banking Risk*

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Abstract

In this paper we analyze the optimal capital requirement in a model of banks with heterogeneous investment risks and asymmetric information. Asymmetric information prevents depositors from charging an actuarially-fair interest rate based on banking risk, and leads to cross-subsidization across banks. A capital requirement in the form of a leverage constraint reduces the investment of riskier banks and partially mitigates the pecuniary externality on deposit rates. We characterize the optimal capital requirement under two different informational assumptions. When the depositors and policymaker have no information about banking risk, only a uniform leverage constraint is possible. In this case, the optimal leverage constraint is tighter than the first-best leverage ratio and strictly improves social welfare. When the depositors and policymaker observe a noisy signal of banking risk, a signal-based leverage constraint is possible. We demonstrate that the optimal signal-based leverage constraint is tighter when the signal has worse precision, rather than a larger level of expected risk.

JEL Classification: G21, G28

Keywords: Capital requirements, Banking regulation, Asymmetric information

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1 Introduction

The 2008 recession is frequently referred to as a financial crisis caused in part by the excessive risk-taking of banks and other financial institutions. How best to regulate financial institutions has motivated various rounds of Basel accords.

The conventional wisdom of setting capital requirement for financial institutions is to base requirements on the risk of the underlying assets on their balance sheet(on Banking Supervision (2017)). However, prior to the 2008 financial crisis, mortgage-backed securities were widely considered safer assets, consistently receiving the highest ratings from all three credit bureaus (Ashcraft et al. (2010)). The financial crisis revealed that these beliefs were incorrect: the risk for these securities was systematically higher than originally thought.

When financial institutions carry assets that are engineered in a complicated fashion, it is difficult for investors and policymakers to accurately assess the risk of these institutions. This motivates our paper, which explores the optimal capital requirement in an environment with various degrees of asymmetric information between financial institutions and investors.

In particular, we construct a model of banks with investment projects featuring heterogeneous levels of risk. Bank risk is only partially observable to depositors and policymakers. Optimal leverage constraints are characterized under different assumptions on the information set of depositors and policymakers.

First, we consider a full information benchmark where depositors accurately observe the risk of banks. In this case, depositors require an interest rate on deposits based on the risk of each bank. These deposit rates incentivize the efficient allocation of deposits to banks and leverage constraints cannot improve on the outcome.

Next, we consider the case with imperfect information. In this case, depositors observe an imprecise signal of the risk of each bank. This signal crudely captures that rating agencies or regulators are only able to inaccurately assess the risk of financial institutions through limited observables such as profitability ratios, liquidity ratios, and debt ratios. Based on signals, depositors are able to make partial inference about the risk of banks. The imprecise inference leads to pooling of the deposit interest rates. As a result, riskier banks pay a deposit rate that is lower than the actuarially fair level, hence they are overly leveraged relative to the efficient level. Safer banks are overcharged for deposits and hence are inefficiently small. In the presence of this pecuniary externality, the deposit rate alone is insufficient to incentivize the efficient allocation of deposits. A role for leverage constraints emerges. Leverage constraints limit the leverage of riskier banks, which are forced to reduce their deposit intake. This results in a lower deposit rate and encourages additional deposit intake by safer banks.

There are two main messages of this paper. First, leverage constraints do not replace the role of market prices (interest rates on deposits) in facilitating the efficient allocation of deposits. Instead, the former supplements the latter when there are market failures, which in this paper, arise due to incomplete information. Second, optimal leverage constraints should be based on the severity of incomplete information (conditional variance of risk), rather than the average risk of banks (conditional mean of risk). More severe incomplete information in the form of nosier signals on the risk of banks leads to greater pooling in deposit rates, hence a larger pecuniary externality. In this case, a stricter leverage constraint is justified.

Our paper contributes to the growing literature that analyzes the effect of capital requirements.[1] Despite the large body of literature on this topic, there has been no consensus in data or in theory about the effects of augmenting capital requirements. Empirically, Tanda (2015) reviews the effect of capital regulation on the capital ratio and risk exposure of banks, and discovers varying results across time, country or class of capital analyzed. Theoretically, VanHoose (2007) reviews studies on bank capital regulation, and finds mixed predictions on asset risk and of soundness of the banking system as a whole.

Closely related to our paper is the logic from Kim and Santomero (1988): improperly priced deposit insurance prevents market prices from reflecting differences in risk across investment options and as a result, banks over-invest in riskier projects. In our model, market failures arise from investments that are inaccurately understood. In both frameworks, banks over-invest in under-priced assets. In contrast to Kim and Santomero (1988), risk based capital requirements do not work in our framework since they can only be implemented based on the imperfect signal rather than the actual level of risk. Pooling of the deposit rate still occurs across banks sharing any particular signal. Our solution is to base leverage constraints on the accuracy of the signal rather than the signal itself.

Most papers in this literature attempt to characterize the optimal level of capital requirements. Begenau and Landvoigt (2016), Diamond and Rajan (2000), and Van den Heuvel (2008) study the optimal capital requirement under the trade-off of financial stability and liquidity. Admati et al. (2013) falsifies the pervasive view that capital requirements negatively affect credit availability causing social inefficiency. DeAngelo and Stulz (2015) demonstrates that higher leverage ratios are optimal for banks, and imposing leverage limits results in inefficiency by impeding banks' ability to compete with shadow banks. Hellmann et al. (2000) points out that capital requirements create inefficiencies in a dynamic model of moral hazard by reducing the gambling incentive of banks. Kim and Santomero (1988) investigates the role of capital regulation when deposit insurance is inefficiently priced. Morrison and White

[1] In this paper capital constraints are implemented in the form of a limit on leverage ratios. A minimum capital requirement is conceptually equivalent to a limit on the leverage ratio.

(2005) analyzes a banking model with adverse selection and moral hazard. Repullo (2004) presents a dynamic model of imperfect bank competition in which capital requirements ensures the existence of a prudent equilibrium. Blum (1999), Koehn and Santomero (1980) and Rochet (1992) deliver a counter-intuitive result where a tighter capital requirement leads to increased risk taking by banks. Davydiuk (2017) focuses on the optimal capital requirement over the business cycle and discovers larger welfare gain from a dynamic capital requirement than a fixed one.

We contribute to this literature by focusing on a different dimension for setting capital requirements; namely, that capital requirements should address the conditional variance of risk, based on the signal that depositors observe, rather than the expected risk level (conditional mean of risk, based on signal).

Our paper is also related to the literature which seeks to understand the effects of macro-prudential capital regulation on banks. Most of this literature focuses on the trade-offs between welfare gains achieved through a reduction in the inefficient risk-taking incentives of limited liability and the welfare losses of reduced lending, output Martinez-Miera and Suarez (2014); Nguyen (2014), and liquidity Begenau (2015); Van den Heuvel (2008). Corbae and D'Erasmo (2014) analyze the impacts of capital requirements on bank failure. Klimenko et al. (2016) find that capital requirements make banks internalize the effect of lending on the loss-absorbing capacity of banking capital. Malherbe (2015) analyzes optimal capital requirements across business cycles. We focus on the role that capital requirements play on the cross-subsidization across banks.

Finally, our paper is also related to the misallocation literature. In our model, banks face quadratic costs, which makes their technology portray decreasing returns to scale. With this environment, even though all banks exhibit the same expected return in their loans, not all lend the same quantity, causing misallocation. Implementing capital requirements interacts with misallocation, and, as in the misallocation literature,[2] can affect total production.

One natural extension to our model is to address the impact on heterogeneous agents, as Hill and Perez-Reyna (2017) and Mendicino et al. (2017) do.

2 Model

There are two types of agents in our model: depositors and banks. Depositors are deep-pocketed and risk-neutral. Each bank has access to an investment technology with a common mean return but different levels of risk. To invest in their technology, banks can either use

[2]For instance, Banerjee and Duflo (2005), Bartelsman et al. (2013), Buera et al. (2011), Hsieh and Klenow (2009), Hsieh and Klenow (2014), Jeong and Townsend (2007), Restuccia and Rogerson (2008)

their own capital endowment or borrow from depositors. Banks are assumed to have limited liability.

In addition to depositors and banks, a benevolent policymaker is able to impose leverage constraints on banks by limiting their leverage ratio.

We characterize the socially efficient allocation of deposits first. Then we analyze the competitive equilibrium outcomes under three different assumptions on the information set of depositors. Initially, a competitive equilibrium with perfect information is analyzed where depositors can accurately observe the risk of all banks. Next we consider the opposite case where banks of all risk levels are indistinguishable to depositors. Finally, an intermediate case is considered where depositors and policymakers receive an imperfect signal on the risk of banks.

2.1 Banks

There is a unit measure of banks indexed by $\rho \in [\rho_L, \rho_H]$. Each bank has access to an investment project. All projects have the same expected return (normalized to 1) but differ in their probability of success. In particular, the investment project of bank ρ is successful with probability ρ. Upon success, the project generates a return of $\frac{1}{\rho}$. Upon failure, no output is produced.[3] Denote the return to bank ρ by $R(\rho)$, then

$$R(\rho) = \begin{cases} 1/\rho & \Pr = \rho, \\ 0 & \Pr = 1 - \rho. \end{cases}$$

Banks with smaller ρ have a lower probability of success but a higher payoff upon success. For analytic tractability, ρ is assumed to follow a uniform distribution $\rho \sim G(\rho) \equiv U[\rho_L, \rho_H]$.

All banks are endowed with banking equity E. Banks can augment their investment by accepting deposits D from depositors at interest rate R^D. Banks are assumed to have limited liability. In the case where the investment project fails, banks are not liable for paying back depositors. Additionally, a convex cost of banking is assumed to generate interior solutions for deposits intake.[4] In particular, it costs banks $c(I) = I^2/2\varphi$ to manage I units of investment. Costs are paid up front to facilitate the project so the actual amount invested in the project is $I - c(I)$. The expected profit for bank ρ is given by

[3]Concern may arise over the nature of risk modeling in that outcomes are discrete (either complete success or total failure). In Appendix A, we consider a generalized model of bank risk in which outcomes are continuous and projects differ in terms of the variance of outcomes. Although this adds to quantitative complexity, qualitative results are unaffected.

[4]This is one of the ways of endogenously determining bank size as discussed in Baltensperger (1980). Mathematically, a convex cost of banking is similar in nature to a decreasing returns to scale investment technology.

$$\begin{aligned}\Pi(\rho) &= \max_{D} \rho \left[\frac{1}{\rho}\left(I - c\left(I\right)\right) - R^{D} D\right] \\ \text{s.t.} &\quad I = E + D.\end{aligned} \tag{1}$$

2.2 First-Best Benchmark

As a benchmark, we analyze the socially efficient allocation of deposits where a benevolent planner perfectly observes the risk of individual banks and dictates to them how many deposits to accept. The planner maximizes social welfare and solves the following problem for each bank ρ: [5]

$$\max_{D} \rho \frac{1}{\rho}\left(\left(E + D\right) - c\left(E + D\right)\right) - R_f D,$$

where R_f denotes the risk-free interest rate. Since all banks have the same expected return, the optimal quantity of deposits is identical for all banks given by

$$D^{FB}\left(\rho\right) = \varphi(1 - R_f) - E, \forall \rho.$$

Next, we analyze the competitive equilibrium outcomes under three different assumptions about the information of depositors. Each competitive equilibrium outcome is compared with the first-best benchmark, and the optimal leverage constraint is characterized under each scenario.

2.3 Scenario 1: Competitive Equilibrium with Perfect Information

Consider a competitive equilibrium where depositors perfectly observe the risk of each bank. Since banks of risk ρ only succeed with probability ρ, depositors will charge an interest rate of $R^D\left(\rho\right) = \frac{R_f}{\rho}$ on deposits. Given the interest rate, the expected profit for bank ρ is

$$\Pi(\rho) = \max_{D} \rho \left[\frac{1}{\rho}\left(\left(E + D\right) - c\left(E + D\right)\right) - R^{D}(\rho) D\right]. \tag{2}$$

Definition 1. A competitive equilibrium with perfect information consists of a deposit rate function $R^D(\rho)$ and a policy function for banks $D(\rho)$ such that:

[5] This setup would be identical if depositors were domestic since the social planner would need to internalize the opportunity cost of foregoing international deposit rate R_f.

1. Given $R^D(\rho)$, $D(\rho)$ maximizes bank's expected profits:

$$D(\rho) = \arg\max_D \rho \left[\frac{1}{\rho} \left((E + D) - c\,(E + D) \right) - R^D(\rho) D \right].$$

2. The deposit rate $R^D(\rho)$ is actuarially fair for each risk type ρ:

$$R^D(\rho) = \frac{R_f}{\rho}.$$

In the competitive equilibrium with perfect information, the amount of deposits accepted by bank ρ is given by

$$D(\rho) = \varphi(1 - R_f) - E.$$

which coincides with the first-best level.

With perfect information, the risk of banks is perfectly priced into deposit rates. The first-welfare theorem applies and the resulting allocation of deposits is efficient. No capital requirement or any other form of government regulation is necessary.

2.4 Scenario 2: Competitive Equilibrium with Indistinguishable Banks

2.4.1 Competitive Equilibrium without Leverage Constraints

We now consider the opposite case where depositors and policymakers are unable to distinguish between the risk of banks. This results in a single deposit market with a rate that makes depositors indifferent between the international return and domestic deposits. This rate reflects the average risk of all banks. Without a constraint on leverage ratios, the competitive equilibrium is defined as follows.[6]

Definition 2. A competitive equilibrium with indistinguishable banks consists of a deposit rate R^D and a policy function for banks $D(\rho)$ such that

[6]Implicitly assumed here is that depositors can only engage in price competition (i.e., offering a deposit rate to all banks), instead of offering bundles of deposit rate and deposit amount to screen banks of different risks. This is the case when the deposit market is competitive and each depositor cannot observe or contract on the amount of deposits accepted by each bank from other depositors.

In many countries where deposit insurance is not present or where deposit insurance premiums are not based on capital adequacy, this assumption appears reasonable. However, this assumption should be taken with caution for the United States. In the US, the Federal Deposit Insurance Corporation (FDIC) charges banks an assessment rate (insurance premium on deposits) based on CAMELS ratings of banks. Capital adequacy is one of the measures in CAMELS rating. Therefore, effectively, FDIC is essentially offering a non-linear price-quantity contract to banks.

1. Given R^D, $D(\rho)$ maximizes a bank's expected profits:

$$D(\rho) = \arg\max_{D} \rho \left[\frac{1}{\rho} \left((E + D) - c\,(E + D) \right) - R^D D \right].$$

2. The deposit rate R^D is actuarially fair:

$$R^D = \frac{R_f}{\bar{\rho}},$$

   where $\bar{\rho} \equiv \frac{\int \rho D(\rho) dG(\rho)}{\int D(\rho) dG(\rho)}$ which is the deposit-weighted probability of repayment.

The following lemmas characterize the competitive equilibrium with indistinguishable banks.

Lemma 1. *The equilibrium leverage ratio is increasing in the risk of banks.*

This lemma follows directly from the first order condition for banks:

$$D(\rho) = \varphi(1 - \rho R^D) - E.$$

The intuition is simple. All banks have the same expected return. However, due to limited liability, banks of risk ρ only pay back depositors in the successful state, which happens with probability ρ. Unlike the case with perfect information where deposit interest rates are perfectly adjusted to reflect the risk ($R^D(\rho) = \frac{R_f}{\rho}$), here a common deposit interest rate R^D applies to all banks due to incomplete information. As a result, riskier banks are effectively paying a lower expected deposit rate than safer banks. Therefore, the former rationally chooses a higher leverage ratio than the latter.

Clearly, the first-best outcome is not achieved in the case of incomplete information. In particular, in the competitive equilibrium, risky banks accept too many deposits and safe banks accept too few. We summarize this result in the following lemma.

Lemma 2. *Denote the first-best level of deposits by D^{FB} and the deposit-weighted average risk of banks by $\bar{\rho} \equiv \frac{\int \rho D(\rho) dG(\rho)}{\int D(\rho) dG(\rho)}$. Then*

$$D(\rho) > D^{FB}, \forall \rho < \bar{\rho}$$

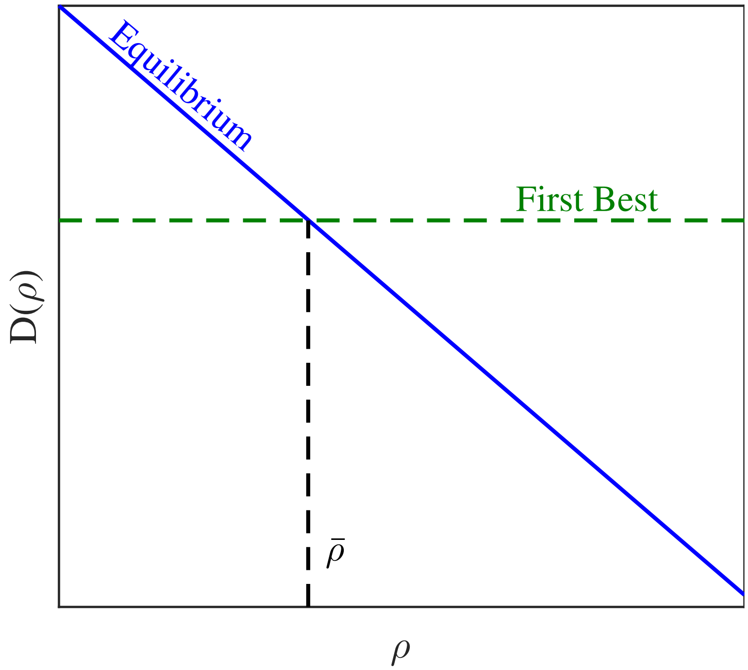
and

$$D(\rho) < D^{FB}, \forall \rho > \bar{\rho}$$

In other words, risky banks borrow too much and safe banks borrow too little, relative to the first-best level.

The proof of this lemma is relegated to Appendix B. The first-best and equilibrium deposits are illustrated in Figure 1.

Figure 1 – Equilibrium v.s. First-Best Deposits



Clearly, the competitive equilibrium outcome is inefficient. Next we introduce a policymaker into the economy and explore whether greater efficiency could be achieved with a leverage constraint.

2.4.2 Competitive Equilibrium with Leverage Constraints

Assume a policymaker has access to a single policy instrument of a leverage constraint λ on banks. The policymaker is assumed to have the same information as depositors: neither observes the risk of banks. A leverage constraint of λ implies $\frac{D}{E} \leq \lambda$ and caps deposits at λE. We now set up and solve a Ramsey problem in order to analyze the optimal leverage constraint.

Definition 3. A Ramsey equilibrium consists of a policy λ representing the leverage constraint, an allocation function of deposits $D(\rho, \lambda)$, and an interest rate $R^D(\lambda)$ on deposits that satisfy the following conditions:

1. Government maximization: the policy λ maximizes social welfare (SW)

$$SW(\lambda) = \max_{\lambda'} SW(\lambda')$$

where

$$SW(\lambda) = \int \left[\rho \frac{1}{\rho} \left(E + D(\rho, \lambda) - c \left(E + D \left(\rho, \lambda \right) \right) \right) - R_f D(\rho, \lambda) \right] dG(\rho)$$

2. For every policy λ', the allocation function $D(\rho, \lambda')$, the interest rate $R^D(\lambda')$, and the policy λ' constitute a competitive equilibrium:

   (a) With a leverage constraint, the bank's problem is modified only slightly in that banks will borrow the lesser of the constraint or their desired amount based on $R^D(\lambda')$. The deposit intake for bank ρ is given by

   $$D(\rho, \lambda') = \min(\lambda' E, D(\rho, 0)) = \min(\lambda' E, \varphi(1 - \rho R^D(\lambda')) - E).$$

   (b) $R^D(\lambda')$ remains an actuarially fair deposit rate based on the deposit-weighted average risk $\bar{\rho}(\lambda') = \frac{\int \rho D(\rho, \lambda') dG(\rho)}{\int D(\rho, \lambda') dG(\rho)}$:

   $$R^D(\lambda') = \frac{R_f}{\bar{\rho}(\lambda')}$$

The following lemma characterizes an important feature of leverage constraint λ.

Lemma 3. *$R^D(\lambda)$ is increasing in λ.*

While we provide the intuition for the lemma here, the proof of this lemma is relegated to Appendix C.

Recall that R^D is the actuarially fair deposit rate based on the deposit-weighted average risk $\bar{\rho}$. Since riskier banks are more leveraged than safer banks (Lemma 2), the leverage constraint is binding for the former and slack for the latter. Now consider a tightening of the leverage constraint λ. Riskier banks with a binding constraint have to reduce the deposits they accept, while safer banks are unaffected. This pushes the deposits at riskier banks down towards the first-best level. Additionally, reduced deposits intake by riskier banks reduces the deposit-weighted average risk $\bar{\rho}$ and leads to a reduction in the actuarially-fair deposit rate R^D. A reduction in R^D encourages safer banks to accept more deposits, which pushes the deposits at safer banks up toward the first-best level as well.

The following lemma characterizes the key result of this section.

Lemma 4. *The optimal leverage constraint λ^* is tighter than $\frac{D^{FB}}{E}$, the first-best leverage ratio.*

$$\lambda^* \in (0, \frac{D^{FB}}{E}].$$

Further, social welfare is strictly higher with a leverage constraint of λ^* than with no leverage constraint at all.

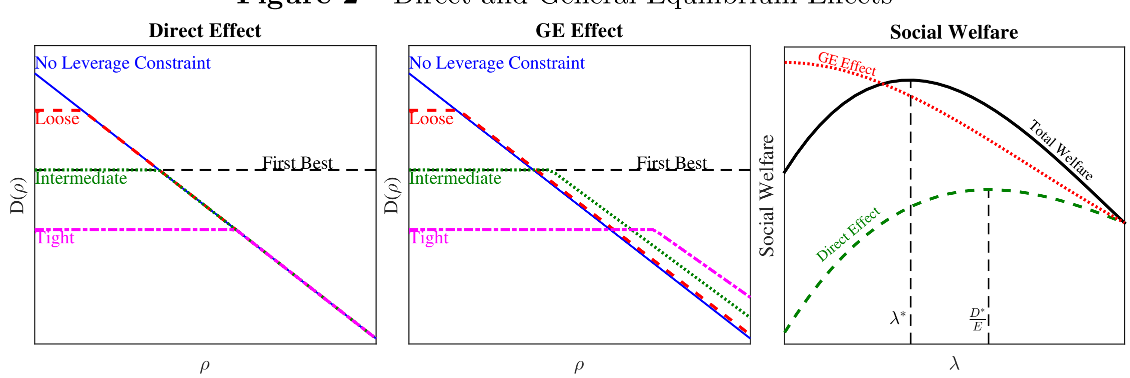
The proof of this lemma is relegated to Appendix D

Conceptually, a tightening of leverage ratios (decrease in λ) exerts two forces on social welfare: a direct effect via the constraint and a general equilibrium effect through the change in deposit rate R^D. These two effects are visually presented in Figure 2.

The direct effect is demonstrated in the left panel of Figure 2. The solid blue line represents the deposit function without any leverage constraint, which is increasing in the risk of banks as demonstrated in Lemma 1. Each leverage constraint is displayed as a horizontal straight line representing a cap on the amount of deposit intake. When the leverage constraint is loose (dashed red line), the constraint only binds for banks with the lowest ρ's (riskiest banks). When the leverage constraint is tightened (dash-dotted purple line), the leverage constraint binds for a larger range of ρ's.

The direct effect of the leverage constraint on social welfare is maximized when $\lambda = \frac{D^{FB}}{E}$. When leverage constraints are relatively loose, a tightening of the constraint (in the figure, from the dashed red line to the dotted green line) results in a direct effect which improves social welfare by constraining risky banks from borrowing too much. As leverage constraints are further tightened (from the dotted green line to the dash-dotted purple line), even risky banks are restricted to borrow less than the first-best amount. After this point, the direct effect starts to reduce social welfare. The welfare implication of the direct effect is also represented by the green dashed line in the right panel of Figure 2, with maximum direct effect achieved at $\lambda = \frac{D^{FB}}{E}$.

Figure 2 – Direct and General Equilibrium Effects



The general equilibrium effect works through the interest rate on deposits R^D and can be observed in the middle panel of Figure 2. From Lemma 3, the deposit rate R^D is lowered as the leverage constraint is tightened. A lower cost of deposits (smaller R^D) incentivizes safer banks to accept more deposits, rotating the deposit line counter-clockwise and generally improving social welfare (dashed red line to dotted green line, then to dash-dotted purple line.) The welfare implication of the general equilibrium effect is represented by the red dotted line in the right panel of Figure 2.

Adding up the direct effect and general equilibrium effect, it is easy to see why the optimal leverage constraint is tighter than the first-best leverage ratio. Start λ at infinity (no leverage constraint) and gradually lower it (tighen the constraint). Initially, both the direct and general equilibrium effects are contributing to social welfare (moving leftwards on the right panel of Figure 2.) This continues until λ hits $\frac{D^{FB}}{E}$. After that, further tightening of λ results in a trade-off between the direct and general equilibrium effects.

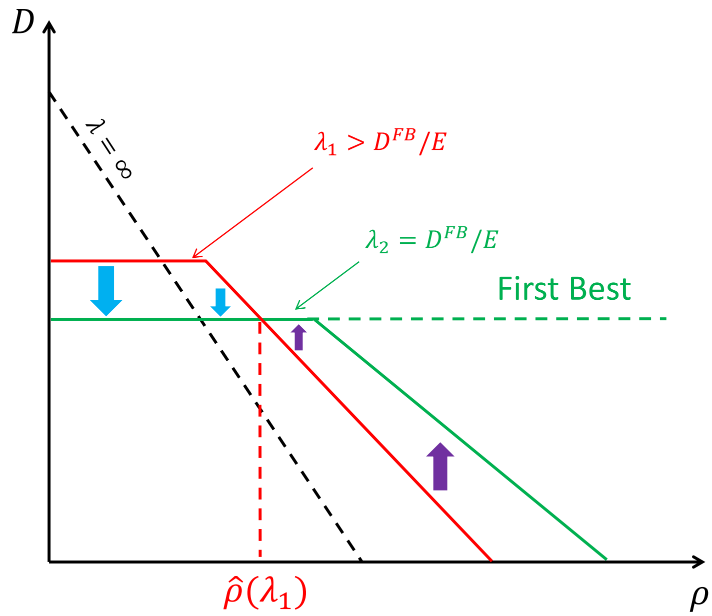
Graphically, it is easy to see why a leverage constraint λ looser than the first-best leverage ratio can never be optimal. Consider any leverage constraint λ_1 with $\lambda_1 > \frac{D^{FB}}{E}$. The corresponding deposit function is represented by the solid red line in Figure 3. Now consider a tightening of the leverage constraint to $\lambda_2 = \frac{D^{FB}}{E}$, with the corresponding deposit function given by the solid green line. This tightening of constraint from λ_1 to λ_2 results in a direct effect and a general equilibrium effect, both tending to push the deposit function towards the first-best level (dotted green line). The direct effect is represented by the blue arrows where riskier banks are forced to reduce leverage. The general equilibrium effect is represented by the purple arrows where safter banks are encouraged to borrow more due to a reduced deposit rate R^D. In summary, the deposit amount for all types of banks are pushed towards the first-best level. Therefore, a leverage constraint looser than the first-best leverage ratio can always be improved upon.

It may be surprising that for this economy, the competitive equilibrium can always be improved by government intervention even though the government observes no more information than depositors. This is because riskier banks are exerting a price externality through the deposit rate and this externality can be corrected by the government.

2.5 Scenario 3: Partial Information in the Form of Noisy Signals on the Risk of Banks

In practice, depositors screen banks or firms according to credit score, loan to value ratio, and other signals which are correlated to risk. These signals provide partial information with

Figure 3 – Visual representation of Lemma 4



respect to the risk of the underlying asset. In this section, we assume that depositors and the policymaker observe imprecise information on the risk of banks. In particular, both parties observe the same noisy signal $\tilde{\rho}$ on bank risk ρ, where $\tilde{\rho} = \rho + \epsilon$, $\epsilon \sim H(\epsilon) \equiv N(0, \sigma^2)$.[7] σ^2 is a parameter reflecting the accuracy of the signal. Scenarios 1 and 2 are special cases of this scenario. In particular, $\sigma = 0$ reduces to the case with perfect information, and $\sigma = \infty$ reduces to the case with indistinguishable banks.

After observing $\tilde{\rho}$, depositors and the policymaker make inference of the underlying risk of banks $\rho|\tilde{\rho}$. Market clearing determines the deposit interest rate $R^D(\tilde{\rho})$ and the policymaker decides what leverage constraint $\lambda(\tilde{\rho})$ to impose.

We now define a Ramsey's problem in order to characterize the optimal leverage constraint.

Definition 4. A Ramsey equilibrium consists of a policy function $\lambda(\tilde{\rho})$ representing a leverage constraint for each signal $\tilde{\rho}$, an allocation function of deposits $D(\rho, \tilde{\rho}; \lambda(\cdot))$, and an interest rate $R^D(\tilde{\rho}; \lambda(\cdot))$ on deposits that satisfy the following conditions:

[7]Signals need not have bounded support. A negative $\tilde{\rho}$ still provides information about the true ρ even though ρ cannot be negative.

1. Government maximization: the policy $\lambda(\tilde{\rho})$ maximizes the social welfare

$$\max_{\lambda'(\cdot)} \int_\rho \int_\epsilon \left[\frac{1}{\rho}\rho\left(E + D\left(\rho, \tilde{\rho}; \lambda'(\cdot)\right) - c\left(E + D\left(\rho, \tilde{\rho}; \lambda'(\cdot)\right)\right)\right) - R_f D\left(\rho, \tilde{\rho}; \lambda'(\cdot)\right)\right] dH(\epsilon) dG(\rho)$$
$$\text{s.t.} \tilde{\rho} = \rho + \epsilon$$

2. For every policy $\lambda'(\tilde{\rho})$, the deposit function $D(\rho, \tilde{\rho}; \lambda'(\cdot))$, the interest rate $R^D(\tilde{\rho}; \lambda'(\cdot))$, and the policy $\lambda'(\tilde{\rho})$ constitute a competitive equilibrium:

   (a) The deposit intake for a bank of risk ρ and signal $\tilde{\rho}$ is given by

   $$D\left(\rho, \tilde{\rho}; \lambda'(\cdot)\right) = \min(\lambda'(\tilde{\rho}) E, \varphi(1 - \rho R^D(\tilde{\rho}; \lambda'(\cdot))) - E).$$

   (b) For each signal $\tilde{\rho}$, $R^D(\tilde{\rho}; \lambda'(\cdot))$ is an actuarially fair deposit rate based on the deposit-weighted average risk $\bar{\rho}(\tilde{\rho}; \lambda'(\cdot)) \equiv \frac{E_\epsilon[\rho D(\rho, \tilde{\rho}; \lambda(\cdot)) | \rho + \epsilon = \tilde{\rho}]}{E_\epsilon[D(\rho, \tilde{\rho}; \lambda(\cdot) | \rho + \epsilon = \tilde{\rho}]}$:

   $$R^D\left(\tilde{\rho}; \lambda'(\cdot)\right) = \frac{R_f}{\bar{\rho}\left(\tilde{\rho}; \lambda'(\cdot)\right)}.$$

Similar to the scenario with no information, banks interact indirectly through the interest rate on deposits. When riskier banks accept more deposits, the interest rate on deposits rises, which discourages safer banks from accepting deposits. In some sense, riskier banks exert a pecuniary externality on safer banks.

Moreover, the deposit interest rate is also the only interaction among banks. Any two banks with different signals $\tilde{\rho}_1$ and $\tilde{\rho}_2$ are charged different deposit rate $R^D(\tilde{\rho}_1; \lambda(\cdot))$ and $R^D(\tilde{\rho}_2; \lambda(\cdot))$. The deposit intake of bank 1 has no impact on the interst rate for deposits of bank 2, hence zero impact on the deposit intake of bank 2. Therefore, the pecuniary externality only exists among banks that have received the same signal $\tilde{\rho}$.

Since the only observable for the policymaker is the noisy signal $\tilde{\rho}$, a uniform leverage constraint has to be imposed across all banks with the same signal $\tilde{\rho}$ regardless of the actual underlying risk ρ.

The observation that both the interest rate on deposits R^D and the leverage constraint λ depend only on the signal $\tilde{\rho}$ greatly simplifies the Ramsey problem. In particular, the optimal leverage constraint for different signals $\tilde{\rho}$'s can be solved independently.

Lemma 5. *$\forall \tilde{\rho}$, the optimal leverage constraint $\lambda(\tilde{\rho})$ in the Ramsey's problem is the solution to*

$$\lambda(\tilde{\rho}) \in \arg\max_\lambda \int_\rho \left[\frac{1}{\rho}\rho\left(E + D(\rho) - c\left(E + D(\rho)\right)\right) - R_f D(\rho)\right] dG(\rho | \tilde{\rho})$$

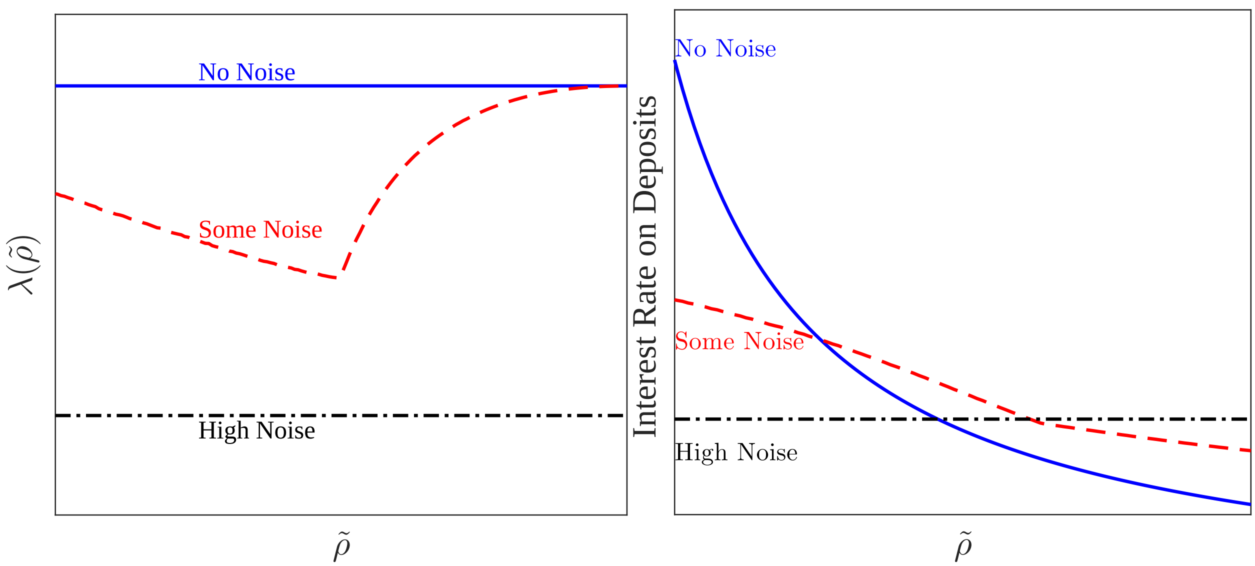
subject to

$$D(\rho) = \min\left(\lambda E, \varphi(1-\rho R^D) - E\right)$$
$$R^D = \frac{R_f}{\bar{\rho}}$$
$$\bar{\rho} \equiv \frac{E_\rho\left[\rho D(\rho)\,|\tilde{\rho}\right]}{E_\rho\left[D(\rho)\,|\tilde{\rho}\right]}$$

In other words, for any signal $\tilde{\rho}$, the optimal $\lambda(\tilde{\rho})$ reduces to setting a uniform leverage ratio for the conditional distribution of risk $\rho|\tilde{\rho}$.

Proof. **trivial.** □

Figure 4 – Optimal Risk-Based Capital Constraint



The left panel of Figure 4 displays the optimal capital constraint for three economies where signals have different accuracy. The solid blue line corresponds to an economy of perfect information in which signals perfectly reveal the risk of banks ($\sigma = 0$). The dash-dotted black line corresponds to an economy of indistinguishable banks in which signals are not informative at all ($\sigma = \infty$). The dashed red line is the intermediate case where signals are informative yet not accurate ($0 < \sigma < \infty$).

Without noise, depositors have perfect information of the risk of banks. As a result, the interest rates on deposits perfectly reflects the risk of banks $R^D(\rho) = \frac{R_f}{\rho}$ and incentivizes the efficient allocation of deposits. This is demonstrated by the solid blue line in the right panel of Figure 4. The competitive equilibrium outcome is efficient without any leverage constraint. Any λ large enough to allow the first-best quantity of deposits is efficient (and the constraint is always slack).

With uninformative signals, depositors have no information of the risk of banks. A common interest rate on deposits is charged (dash-dotted black line, right panel of Figure 4). With no information on bank type, the pecuniary externality is the most severe, which calls for the tightest leverage constraint (dash-dotted black line, left panel of Figure 4). Since the policymaker has the same information as depositors, a uniform and tight leverage constraint is imposed.

With partially informative signals, interest rates are calculated based on the inferred risk of signals $(\rho|\tilde{\rho})$, which only partially reflect the risk of banks (red dashed line, right panel of Figure 4). Riskier banks are charged interest rates that are actuarially too low and safer banks charged rates which are too high. Leverage constraints complement the role of interest rates in capping the deposit intake by riskier banks. The optimal leverage constraints (red dashed line, left panel of Figure 4) lies in between the no-noise case and the infinite-noise case.

One interesting observation is that the optimal leverage constraint $\lambda(\tilde{\rho})$ is not monotonically increasing in the signal $\tilde{\rho}$. This seems counter-intuitive because higher signals communicate lower expected risk $E[\rho|\tilde{\rho}]$ (safer banks) which seems to justify more relaxed leverage limits. The reason for this counter-intuitive result is most easily explained through an example.

Consider two signals, the first being precise and the second being imprecise.[8] A precise signal (low $Var[\rho|\tilde{\rho}]$) implies that the group of banks receiving the signal have similar levels of risk. Therefore, when depositors observe this precise signal, they are able to accurately infer the risk of the banks. This allows them to charge an appropriate interest rate on deposits which naturally incentivizes banks to leverage near the optimal amount. In this case, the outcome is fairly close to the first best and a tight leverage constraint cannot do much to improve outcomes.

However, if a signal is imprecise (high $Var[\rho|\tilde{\rho}]$), the signal represents a diverse range of banks. Depositors charge an interest rate based on the average risk of all the banks that yield the same signal, which leads to significant cross-subsidization across banks. Banks will borrow too much if they are riskier than $E[\rho|\tilde{\rho}]$ and too little if they are safer than expected. In this case a tightened leverage constraint can improve outcomes by restricting riskier banks from borrowing too much and lowering the resulting deposit rate, incentivizing

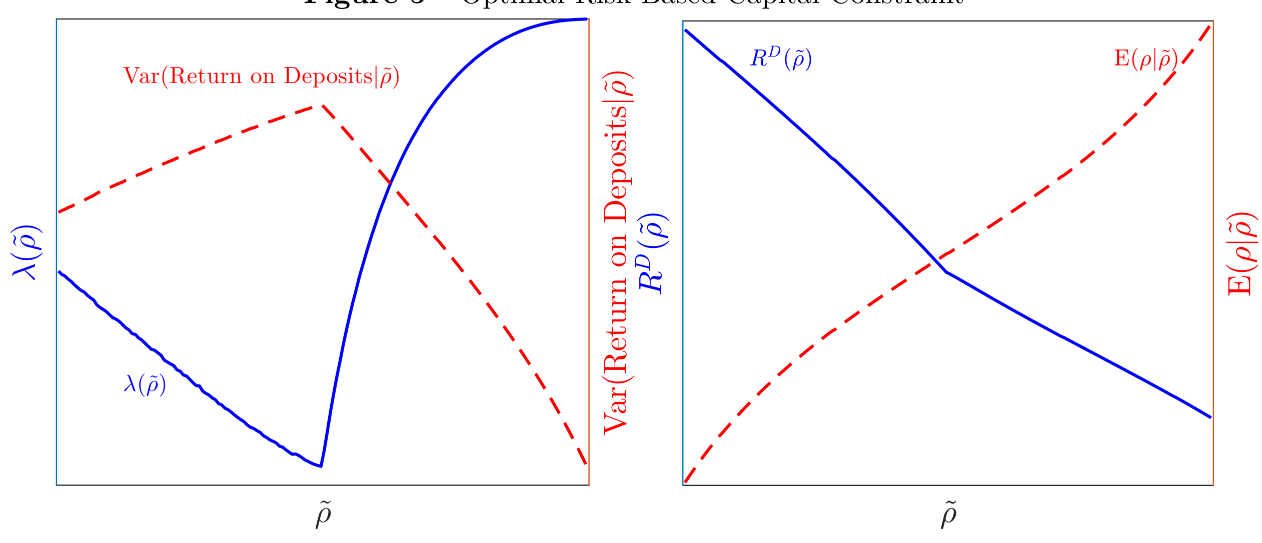
[8] Precision of the signal can be measured in different ways. The relevant measure for depositors is the conditional variance on returns, $Var\left[\left(R_f - R^D(\tilde{\rho})\right)D(\rho)|\tilde{\rho}\right]$, which is used in Figure 5. We abuse notation slightly by writing the less cumbersome $Var[\rho|\tilde{\rho}]$ throughout the narative to stand for the precision of the signal.

safer banks to move towards the first best deposit amount. Therefore, the setting of leverage constraints should be primarily based on the precision of the signal, rather than the expected risk implied by the signal. The latter will be incentivized via the interest rate.

The left panel of Figure 5 highlights the relationship between the optimal leverage constraints $\lambda(\tilde{\rho})$ and the dispersions of the signal $Var[\rho|\tilde{\rho}]$ over the signal range. The red dashed line is a measure of the dispersion of the signal.[9] Since ρ is assumed to be uniformly distributed, extreme signals $\tilde{\rho}$ close to the lower and upper bounds of the distribution of ρ are the most informative (i.e. low $Var[\rho|\tilde{\rho}]$), while signals $\tilde{\rho}$ in the middle are less informative (i.e. large $Var[\rho|\tilde{\rho}]$). This explains why the signal dispersion displays an inverse-U shape. The optimal leverage constraints are represented by the solid blue line. The optimal constraints are loose towards the ends when signals are precise, and are tighter around the center when signals are imprecise.

In the right panel of Figure 5 we display the relationship between interest rates and the expected risk. A larger signal $\tilde{\rho}$ suggests that the banks underlying the signal are safer on average. The resulting interest rates are therefore lower for larger signals. Leverage constraints do not replace the role of market prices (deposit rates) in facilitating the allocation of resources. Instead, the former complements the latter in the case of market frictions, such as unobservable risk.

Figure 5 – Optimal Risk-Based Capital Constraint



[9]Here the dispersion of the signal is defined to be $Var\left[\left(R_f - R^D\left(\tilde{\rho}\right)\right)D\left(\rho\right)|\tilde{\rho}\right]$, which measures the variance on the returns of depositors. Note that the conditional mean $E\left[\left(R_f - R^D\left(\tilde{\rho}\right)\right)D\left(\rho\right)|\tilde{\rho}\right] = 0$ from the definition of R^D.

3 Conclusion

In this paper, we characterize the optimal leverage constraint in a model where banks feature heterogeneous levels of risk which are only partially observable to depositors and policymakers.

In our model, banks accept deposits and invest them in projects with differing levels of risk. Depositors lack information about the risk level of each individual bank and instead charge a deposit rate based on the overall risk portfolio of all banks. As a result, banks with higher risk are paying an interest rate that is actuarially too low, which incentivizes these banks to invest above the socially optimal level. On the contrary, banks with lower risk are paying an interest rate that is actuarially too high. These safer banks are investing below the socially efficient level of investment. This generates a loss of social efficiency.

Optimal capital requirements are characterized under three different assumptions on the information set of depositors and policymakers. First, we consider a full information benchmark where depositors and policymakers perfectly observe the risk of each bank. In this case, the interest rate on deposits incentivizes optimal allocations as the conditions for the first welfare theorem hold. No leverage constraint is necessary.

Second, we consider the opposite case where depositors and policymakers have no information on the risk of banks. With no information, a uniform deposit interest rate is charged on all banks, leading to a failure of the conditions for the first welfare theorem. A leverage constraint is able to improve on social welfare by restricting the deposit intake of risky banks and encouraging the deposit intake of safer ones. Our primary finding for this setup is that social welfare is not monotone in the tightness of the leverage constraint due to a trade off between a direct effect and a general equilibrium effect. The optimal leverage constraint has an interior solution and is always tighter than the leverage ratio of the first-best allocation.

Lastly, we consider an intermediate case where depositors and policymakers observe a noisy signal of the risk of banks which provides imperfect information about the underlying risk. In this case, a separate leverage constraint can be designed for each possible signal. Surprisingly, under the optimal policy, the tightest leverage constraint is not imposed when the expected risk of banks is the highest. Instead, it is imposed when the expected risk of banks has the highest inaccuracy or dispersion. The deposit interest rate and the leverage constraint specialize in different roles for the maximization of social welfare. The former focuses on the conditional mean of risk and the latter focuses on the conditional dispersion.

To summarize, even in the case of limited liability, heterogeneity in risk is not a sufficient condition for market failure and differences in prices alone can achieve an efficient allocation of societal resources. When information about risk heterogeneity is asymmetric, then market failures can arise and limits on leverage ratios are able to improve market outcomes. This has direct application in the banking sector but may also be applied more broadly.

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Appendix

A Generalizing our risk setup

In this section we extend the setup of the banks to account for risk along a different dimension: instead of characterizing banks by the probability of the success in their projects, we now characterize them by the variance in their losses. We will highlight that the main mechanism in our model still holds; namely, cross-subsidization across banks.

The domestic economy consists of a unit measure of banks indexed by $\sigma \sim F[0, \sigma_H]$. Banks have access to an investment project with risky return. All projects have the same expected return (normalized to 1) but differ in variance of outcomes. Actual outcomes, denoted by R, follow a distribution $G(\cdot;\sigma)$ with variance σ^2 and mean 1. Banks with smaller σ have smoother outcomes and banks with larger σ have more variant outcomes.

Banks are restricted to invest in their own project. All banks are endowed with banking equity E and can augment their investment by accepting deposits $D(\sigma)$ from international depositors at interest rate R^D. We still make two key assumptions with respect to banks. First, banks are limited in their liability. Second, banks of different risk levels are indistinguishable from the perspective of depositors. As a result, depositors charge a common deposit rate R^D to all banks which reflects the overall risk of the entire economy.

We also assume that banks pay a convex cost $c(I) = I^2/(2\varphi)$ to manage I units of investment. As before, costs are incurred before returns on investment are realized. The expected profit for any bank is given by:

$$\Pi(\sigma, D) = \int \max\big(R(E + D - c(E + D)) - R^D D, 0\big)\, dG(R;\sigma) - E.$$

Let $\underline{R} = R^D \frac{D}{E+D-c(E+D)}$ be the smallest realization of the project for which the bank does not default on the loan. We can rewrite profits as:

$$\Pi(\sigma, D) = \int_{\underline{R}}^{\infty} R(E + D - c(E + D))dG(R;\sigma) - (1 - G(\underline{R}))R^D D - E.$$

The depositor receives $R^D D$ if $R \geq \underline{R}$ and $R(E + D - c(E + D))$ if $R < \underline{R}$.

Given this environment, a competitive equilibrium is given by

1. Given R^D, $D(\sigma)$ maximizes bank's expected profits.

$$D(\sigma) = \arg\max_D (E + D - c(E + D)) \int_{\underline{R}}^{\infty} R dG(R;\sigma) - (1 - G(\underline{R}))R^D D - E$$

2. Deposit rate R^D is actuarially fair.

A given bank repays in expectation:

$$\hat{R}(\sigma; R^D) = \frac{\int_0^{\underline{R}} R(E + D - c(E + D))dG(R;\sigma) + R^D D(1 - G(\underline{R};\sigma))}{D}$$

The expected return across all banks must equal the risk free rate.

$$\int \hat{R}(\sigma; R^D)dF(\sigma) = R_f$$

The solution to the problem of bank σ is implicitly given by

$$R^D = (1 - c'(D+E)) \int_{R \geq \underline{R}} \frac{R}{1 - G(\underline{R}; \sigma)} dG(R; \sigma)$$

where

$$\underline{R}(D) = R^D \frac{D}{E + D - c(E+D)}.$$

Let $g(R;\sigma) \equiv \frac{\partial G}{\partial R}(R;\sigma)$. As long as

$$\begin{aligned}
&\left[c''(D+E) \int_{R \geq \underline{R}} \frac{R}{1 - G(\underline{R}; \sigma)} dG(R; \sigma)\right. \\
&\left.\frac{1 - c'(D+E)}{(1 - G(\underline{R}))^2} g(\underline{R}) R'(D) \left[(1 - G(\underline{R}))\underline{R} - \int_{R \geq \underline{R}} R dG(R; \sigma)\right]\right] \\
&\times \frac{1 - c'(D+E)}{1 - G(\underline{R})} \int_{R \geq \underline{R}} R \frac{\partial g}{\partial \sigma}(R; \sigma) dR > 0
\end{aligned}$$

then $D^*(\sigma)$ is increasing in σ. The condition above is a technical condition that is satisfied by a normal distribution under plausible parameters.

The intuition behind $D^*(\sigma)$ being increasing in σ is fairly straightforward: since all banks face the same deposit rate and banks have limited liability, less risky banks (lower σ) subsidize riskier banks. As long as this condition holds, our main mechanism remains in tact.

B Proof of Lemma 2

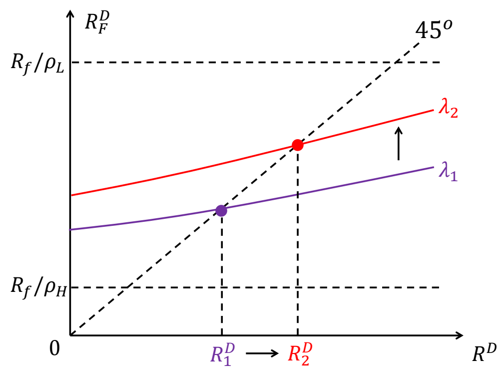
Proof. Deposit rates are actuarially fair, $R^D = \frac{R_f}{\bar{\rho}}$. Therefore, $D(\bar{\rho}) = \varphi(1 - \bar{\rho}R^D) - E = \varphi(1 - R_f) - E = D^*$. In other words, any bank with risk $\rho = \bar{\rho}$ will choose the first-best level of deposits, D^*. The rest of the proof follows from Lemma 1 which states that deposits are decreasing in ρ. □

C Proof of Lemma 3

Proof. Deposit rate $R^D(\lambda)$ is set to be acturially fair, solving the following equation:

$$R^D = \frac{R_f}{\bar{\rho}} = \frac{R_f}{\frac{\int \rho D(\rho) dG(\rho)}{\int D(\rho) dG(\rho)}} = R_f \frac{\int D(\rho; R^D) dG(\rho)}{\int \rho D(\rho; R^D) dG(\rho)}.$$

Figure 6 – R^D as a Solution to a Fixed-Point Problem



This is a fixed point problem and we will focus on the more interesting case where solutions exist.

It is possible that there are multiple solutions of $R^D(\lambda)$ which satsify the above condition. Suppose there are multiple equilibrium deposit rates $R_1^D, R_2^D, \ldots R_N^D$, where $R_1^D < R_2^D \ldots < R_N^D$. We argue that R_1^D is the only deposit rate that will prevail in the deposit market. Other deposit rates are less reasonable due to the following: Consider another equilibrium deposit rate $R^D > R_1^D$. Depositors break even under both deposit rates and hence are indifferent betwen the two. However, bank profits are strictly higher for all banks at the lower deposit rate R_1^D. In this case, one banker can propose to the depositors and recommend them to charge R_1^D instead. If depositors follow the proposal, they will attract all banks since every bank strictly prefers the lower rate R_1^D than the prevailing rate R^D. Moreover, depositors will break even under R_1^D once banks reoptimize how many deposits to accept under this new deposit rate. The proposing bank can even make a voluntary transfer of ε to depositors to make them strictly better off under the switch. In light of this, we suggest when multiple equilibrium deposit rates exist, only the lowest one is the reasonable rate to consider.

Next, we prove that the lowest equilibrium rate $R^D(\lambda)$ is increasing in λ. This is most easily explained in a graphical manner. We will prove it mathematically as well. Fix any λ, R^D that solve the fixed point problem

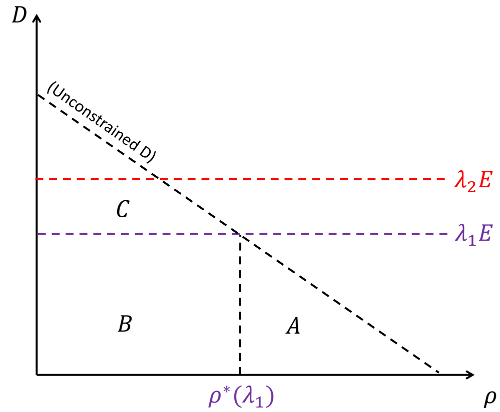
$$R^D = R_f \frac{\int D(\rho; R^D) dG(\rho)}{\int \rho D(\rho; R^D) dG(\rho)}.$$

Graphically, R^D is the intersection of the 45 degree line and the function

$$H\left(R^D\right) = R_f \frac{\int D(\rho; R^D) dG(\rho)}{\int \rho D(\rho; R^D) dG(\rho)}.$$

There are a few properties of the function $H\left(R^D\right)$. First of all, the function has a

Figure 7 – $\bar{\rho}(\lambda)$ Is Increasing in λ



y-intercept that is above the 45 degree line, since

$$H\left(0\right) = R_f \frac{\int D(\rho;0)dG(\rho)}{\int \rho D(\rho;0)dG(\rho)} \geq R_f \frac{\int D(\rho;0)dG(\rho)}{\int \rho_H D(\rho;0)dG(\rho)} = \frac{R_f}{\rho_H} \geq R_f > 0.$$

Second, the function is continuous in R^D. This is because the deposit function D is continuous in R^D and the distribution of ρ is also continuous. Lastly, as λ increases (capital requirement relaxes), $H\left(R^D\right)$ shifts upward, i.e.,

$$H\left(R^D;\lambda_1\right) \leq H\left(R^D;\lambda_2\right), \forall \lambda_1 < \lambda_2.$$

To see this, note that

$$H\left(R^D;\lambda\right) = \frac{R_f}{\bar{\rho}\left(\lambda\right)}.$$

Therefore, it suffices to show that

$$\bar{\rho}\left(R^D;\lambda_1\right) \geq \bar{\rho}\left(R^D;\lambda_2\right), \forall \lambda_1 < \lambda_2.$$

This is easy to demonstrate graphically which we do in Figure 7. Initially, deposits are represented by areas A and B. After relaxing λ, deposits also include area C. Each of these areas has a deposit weighted $\bar{\rho}$. It is straightforward to show that $\bar{\rho}(C) < \bar{\rho}(B) < \bar{\rho}(A)$. $\bar{\rho}(\lambda_1)$, the deposit weighted average ρ when $\lambda = \lambda_1$ can be decomposed into $\bar{\rho}(\lambda_1) = \frac{A}{A+B}\bar{\rho}(A) + \frac{B}{A+B}\bar{\rho}(B)$, this is mathematically expressed in Equation **??**. Since this is an average, $\bar{\rho}(A) > \bar{\rho}(\lambda_1) > \bar{\rho}(B)$. Similarly, $\bar{\rho}(\lambda_2)$ can be decomposed into $\bar{\rho}(\lambda_2) = \frac{A+B}{A+B+C}\bar{\rho}(\lambda_1) + \frac{C}{A+B+C}\bar{\rho}(C)$. Since $\bar{\rho}(C) < \bar{\rho}(\lambda_1)$ and $\bar{\rho}(\lambda_2)$ is an average, then $\bar{\rho}(C) < \bar{\rho}(\lambda_2) < \bar{\rho}(\lambda_1)$.

Mathematically, consider any capital requirement of λ_1. Note that deposit $D\left(\rho;\lambda_1\right)$ is weakly decreasing in ρ:

$$D(\rho) = \min(\lambda_1 E, \varphi(1-\rho R^D) - E).$$

Therefore, there exsits a cutoff level of risk $\rho_1^* \equiv \frac{\varphi-(1+\lambda_1)E}{R^D\varphi}$, below which banks are leveraged to the leverage limt λ_1, and above which the leverage constraint is slack. With ρ_1^* defined in this fashion,

$$
\begin{aligned}
&\bar{\rho}\left(R^D;\lambda_1\right)\\
=&\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\\
=&\frac{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\frac{\int^{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}+\frac{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\\
=&\left[\frac{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]\frac{\int^{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}+\left[1-\frac{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\\
=&\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]-\left[\frac{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}-\frac{\int^{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]\\
\geq&\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]-\left[\frac{\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}\right]\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_1\right)dG}-\frac{\int^{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]\\
=&\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG}\right]-\left[\frac{\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}\right]\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG}-\frac{\int^{\rho_1^*}\rho D\left(\rho;\lambda_1\right)dG}{\int^{\rho_1^*}D\left(\rho;\lambda_1\right)dG}\right]\\
\geq&\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG}\right]-\left[\frac{\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG+\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}\right]\left[\frac{\int_{\rho_1^*}\rho D\left(\rho;\lambda_2\right)dG}{\int_{\rho_1^*}D\left(\rho;\lambda_2\right)dG}-\frac{\int^{\rho_1^*}\rho D\left(\rho;\lambda_2\right)dG}{\int^{\rho_1^*}D\left(\rho;\lambda_2\right)dG}\right]\\
=&\bar{\rho}\left(R^D;\lambda_2\right)
\end{aligned}
$$

for all $\lambda_2 > \lambda_1$.

With these 3 properties of H, now it is easy to see why R^D is increasing in λ. Consider 2 values of λ: $\lambda_1 < \lambda_2$. Let R_1^D and R_2^D be the equilibrium deposit rate under λ_1 and λ_2, respectively. Since $H\left(R^D;\lambda_2\right)$ lies above $H\left(R^D;\lambda_1\right)$,

$$H\left(R_1^D;\lambda_2\right) \geq H\left(R_1^D;\lambda_2\right) = R_1^D.$$

Moreover, $\forall R^D < R_1^D$,

$$H\left(R^D;\lambda_2\right) \geq H\left(R^D;\lambda_1\right) > R^D$$

where the second inequality follows because $H\left(\cdot;\lambda_1\right)$ has a y-intercept that is above the 45 degree line and R_1^D by definition is the first point of intersection between $H\left(\cdot;\lambda_1\right)$ and the 45 degree line.

Therefore,

$$\forall R^D < R_1^D : H\left(R^D;\lambda_2\right) \geq H\left(R^D;\lambda_1\right) > R^D$$
$$H\left(R_1^D;\lambda_2\right) \geq R_1^D.$$

This, combined with the fact that the solution to

$$H\left(R^D;\lambda_2\right) = R^D$$

exists and is denoted by R_2^D, imply that

$$R_2^D \geq R_1^D.$$

□

D Proof of Lemma 4

There are no externalities in this model and agents are risk neutral. Consequently, aggregate social welfare can be expressed as the sum of the expected social welfare contributed by each bank. For any individual bank, social welfare is given by

$$SW(\rho, D) = D + E - c(D + E) - R^f D.$$

Social welfare is maximized at $D^*(\rho) = \varphi(1 - R_f) - E$ and is increasing as D moves towards D^* from either direction. The idea of this proof is that for any $\lambda_1 > \lambda_2 \equiv \frac{D^*}{E}$, deposits are weakly further from D^* under λ_1 than under λ_2 for all banks. This implies that expected social welfare contributed by each bank is weakly less under λ_1 compared to λ_2. This is graphically demonstrated in Figure 3. Mathematically, from Lemma 1, leverage ratios are weakly increasing in ρ. This implies there is a cutoff $\hat{\rho}(\lambda_1)$ such that for all $\rho < \hat{\rho}(\lambda_1)$, $D(\rho, \lambda_1) > D^*$. Lemma 3 implies that $R^D(\lambda_1) > R^D(\lambda_2)$. Note that for any unconstrained bank, D is decreasing in R^D which means that any unconstrained bank under λ_2 would prefer to accept more deposits than they would have under λ_1. Consequently, all banks with $\rho < \hat{\rho}(\lambda_1)$ have moved to D^* under λ_2 since they wish to accept at least as many deposits as they did under λ_1 but are constrained by λ_2 to borrow D^*. For banks with $\rho > \hat{\rho}(\lambda_1)$ these banks were previously borrowing less than D^* under λ_1. Per the discussion above, all unconstrained banks will prefer to accept more deposits under λ_2 unless they become constrained which means all banks will move towards D^* and none will pass it. Consequently, every bank will move weakly towards the optimal deposit amount which results in social welfare being weakly improved. Finally, the unconstrained scenario is identical to any economy where λ is set sufficiently high that it is not binding for any bank. Setting λ_1 to any sufficiently high λ, the logic above demonstrates that social welfare must also be weakly improved under the scenario of no leverage constraint.