[PRELIMINARY AND INCOMPLETE]
Financial (in)stability in Chile*

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Abstract

We develop a comprehensive model of small open economy with a heterogenous real and banking sector, and endogenous default, that allows us to study financial stability under the presence of financial frictions and real economy shocks. In this framework we provide theoretical and empirical evidence of the interplay of real and financial economy. We find that financial frictions - mainly the existence of a default channel - considerably reinforce the impact of commodity prices and exchange rates fluctuations on aggregate output and financial stability and are sufficient to explain dynamics produced during recent fragility periods. Furthermore, we show that the distribution of firm default rates results in a distinct transmission mechanism from real shocks to financial instability across the distribution of bank size. Idiosyncratic shocks to smaller banks permeate the economy via their contagious effects through large systemically important banks.

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1 Introduction

In 2018 the Chilean economy is recovering after a period of slow macroeconomic activity in 2014-2016. The main global economic and geopolitical risks have materialised in lower copper prices. Given its mandate of price and financial stability [1], the Central Bank of Chile (CBC) is continuously evaluating the effectiveness of its policy measures. In particular, the monetary authority implements inflation targeting and is endowed with a set of macro-prudential policy tools. Thus, a relevant question is about the transmission of monetary and prudential policies and the impact of various shocks coming from the international economy, such as copper prices. Moreover, a steadily more exigent macroprudential standard should be taken into account in the policy stance. Indeed, the Chilean Congress is discussing - and prompt to pass - the biggest Banking Law reform of the last decade that considers explicitly the scope and implementation of new regulatory capital requirements that distinguish small and systemic banks and different phases of the financial cycle. However, to the best of our knowledge, there is no specific literature discussing the principles of the optimal monetary policy in Chile in connection with the existence of macroprudential regulation derived from the convergence to international standards, such as Basel III.

The optimal interaction between macroprudential and monetary policies still remains an important challenge globally for policy. (Nachane et al. (2006); Ghosh (2008); Gavalas (2015); Gambacorta and Shin (2016)) show that the more restrictive the rules (in particular, capital requirements), the more contractionary effect the monetary policy may have. In this sense it is non-surprising that the loan portfolios of small banks that have smaller capital adequacy ratios may respond more severely to the contractionary monetary policy impulses (Aiyar et al. (2014); De Marco and Wieladek (2015)). However, strict macroprudential regulation may have an opposite effect on banks' risk-taking. Gale (2010) suggests that too restrictive capital requirements may encourage banks to take higher risks in order to earn higher expected profits. In this case when monetary authorities increase interest rates this may not have a contractionary effect on credit market and the banks will form highly risky loan portfolios as costs of funding increase. As a result, defaults of the risky firms may create the threat to financial stability. It is also worth noting that not only macroprudential regulation has an impact on the monetary transmission mechanism. According to (Borio and Zhu (2012); de Moraes et al. (2016)), the stance of monetary policy may itself affect the optimal level of macroprudential regulation.

The CBC possesses an important set of models for the Chilean economy. These models

[1] Financial Stability is required for Payments Systems continuity, the specific mandate that is in the Organic Constitutional Law.

follow a New-Keynesian approach. Medina and Soto (2007), present a version of the model in a small open economy setting. This explains the business cycles that occurred in the Chilean economy from 1987 to 2005. The frictions included in the model are nominal rigidities and a series of shocks affecting consumption, trade and net external positions. Recently, in order to incorporate the financial sector in a more explicit fashion for the Chilean economy model, García-Cicco and Kirchner (2015) have tested combinations of a simplified version of Medina and Soto (2007), with Gertler and Karadi (2011) and Bernanke et al. (1999). These models include nominal rigidities and consider that the primary source of financial frictions is the presence of asymmetric information as it is manifested in costly state verification and moral hazard. The first paper includes moral hazard in the bank-client relationship, assuming the banker can divert some resources, whereas the second work considers costly state verification of loan/project performance. They have achieved a reasonably good fit of the Chilean economic data moments. In order to explore an alternative approach, and complement the views expressed in García-Cicco and Kirchner (2015), we present a model based on De Walque et al. (2010) which in turn is based on the static analysis of financial (in) stability of Goodhart et al. (2017), Goodhart et al. (2006) and Goodhart et al. (2013).

2 NK Model for the Chilean Economy

Our model includes default and liquidity constraints as the main financial frictions. Thus, the banks, firms and households can default on their obligations subject to default penalties set by the regulator. In this setting, default emerges as an equilibrium phenomenon in that the agents equalize the marginal utility of defaulting with the marginal dis-utility of the penalty. Thus, the purpose of the default penalty is twofold: induce debtors to repay their obligations or refrain from making promises that they will not fulfill. A detailed analysis about how to extend the model of general equilibrium to allow for endogenous default is described in Dubey et al. (2005) and Goodhart et al. (2006). In addition, our setting includes cash in advance constraints as a proxy of liquidity. This approach is adopted from Goodhart et al. (2006). This liquidity constraint prevents the agents from using immediately the proceeds from the sales of their assets; therefore, they have to borrow money from the banking sector for their purchases. Thus, our way of introducing money in the economy is somewhat different to that used by De Walque et al. (2010), because in line with Martinez and Tsomocos (2016) we explicitly model and consider this financial friction. Our results suggest that liquidity and default in equilibrium should be studied contemporaneously due to their interconnectedness and welfare effects. Moreover, agent heterogeneity is essential for assessing the distributional effects of exogenous shocks, since these depend primarily on the part of the economy directly affected. In addition, the presence of financial frictions underlines the importance of studying the impact of shocks to the short as well as the medium-run behavior of financial variables and welfare.

The benchmark model is a small open economy RBC model with a perfectly competitive banking sector and financial frictions. The model economy is populated by households, capital producers, wholesale, intermediate, and final goods firms, copper-extracting firms, systemically important (big) and small banks, and the Central Bank (responsible for monetary and macroprudential policy). The possibility of endogenous default in wholesale producers and the presence of bank micro-prudential regulation are the two financial frictions of the model. Real frictions include investment, capital and assets (liabilities for small banks) adjustment costs. Endogenous default is important because it allows to model risk taking behaviour by firms justifying banking regulation by the Central Bank. Chile is a small open copper exporting economy. A negative shock to the copper price depreciates the exchange rate, raising the price of foreign-priced goods and lowering the demand for domestically-priced goods. Firms subsequently reduce their demand for labor and unemployment increases.

The numeraire will be taken to be the nominal price of final domestically-priced goods, though we will also examine inflation through a an inflation index to provide comparisons with data.

2.1 Circular Flow of Funds

- Wholesale producers of domestically priced goods require funding to invest in physical capital in order to produce domestically priced goods which they sell to intermediate goods producers. They use capital and labor to produce wholesale goods. Unsecured loans are repaid next period, but are defaultable. Secured borrowing is subject to a collateral constraint. They receive equity from Saver households.

- Intermediate producers manufacture a differentiated domestically-priced good which they, in turn, sell them to final goods producers. Intermediate are monopolistically competitive produces set prices for intermediate goods a-la Calvo.

- Domestically-priced Final Goods producers combine the output of intermediate goods producers and sell the composite output at competitive prices domestically and abroad. As the production of these goods occurs domestically and depend on Intermediate producers' prices, these goods are termed 'domestically-priced final goods'.

- Foreign-priced Final Goods are endowed to Saver households as an exogenous process. These goods are traded at foreign prices times the domestic exchange rate (there is full exchange rate pass through). As the goods price is set abroad, these goods are termed 'foreign-priced final goods'.

- Domestically-priced and foreign-priced final goods are differentiated by households but are collectively referenced as 'final goods'.
- Copper extracting firms extract copper using final goods and export it abroad. There is no domestic demand for it.
- Capital producers operate in perfectly competitive markets. They purchase undepreciated capital from Wholesale Firms and combine them with final goods in order to produce new capital. Production is subject to an investment adjustment cost.
- Banks combine Saver households' deposits with their retained earnings and lend them to wholesale firms, Borrower households and other banks on the inter-bank market. Micro-prudential (Basel) regulation requires all banks to hold a certain amount of capital as a proportion of their risk-weighted assets.
- Saver Households own all firms and banks in the economy except for copper producers. They save at both types of banks, pay taxes to the Government, consumes final goods, supplies labor and invests equity in wholesale firms, big banks and small banks.
- There is an agency conflict between the unmodelled managers of big banks and the Saver Household shareholders which results in the objective function of big banks being concave in profits.
- Borrower Households take loans from big and small banks. They consume final goods and supply labor to wholesale firms.
- The Government requires final goods, for spending, receives taxes from Saver households and give transfers to Borrower households.
- The Central Bank regulates banks and sets the interbank interest rate.

2.2 OLG Structure of Firms and Banks

2.3 Endogenous Default

In our model firms issue unsecured debt to banks. Banks therefore have a limited claim on the existing wealth of the borrower and cannot invoke bankruptcy proceedings. Thus a key feature of the paper is that the possibility of default in equilibrium exists on unsecured debt.

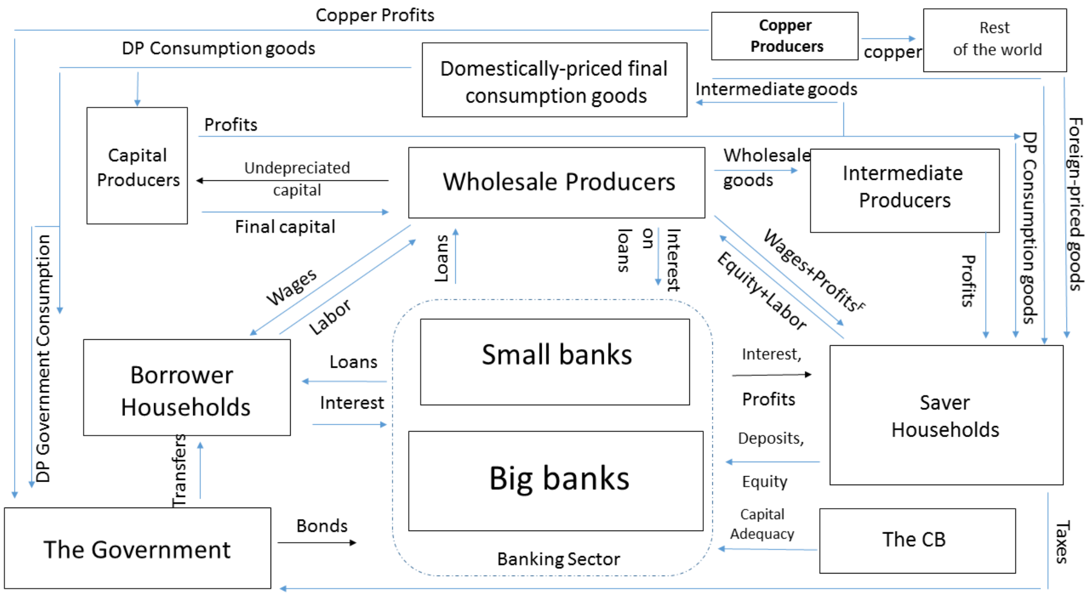

Figure 1: Circular Flows Diagram

We assume that firms can only issue non-state-contingent nominal bonds to banks, or, equivalently, nominally riskless loans are obtained from banks. Firms may choose to renege on some of their debt obligations, but then suffer a renegotiation cost proportional to the scale of default. As firms vanish after their second period of life, their ability to liquidate assets and pay dividends to shareholders is predicated on sucessfully negotiating their existing debt burden. In this sense, the decision to default is strategic.

This cost effectively creates a borrowing constraint and stems from Shubik and Wilson (1977) and Dubey et al. (2005) and applied in Tsomocos (2003), Goodhart et al. (2005) and Goodhart et al. (2006). In the RBC literature, our model shares similar features to De Walque et al. (2010). Our closest methodological precursors are Peiris and Tsomocos (2015) (which studies a two period large open international economy with incomplete markets and default); Goodhart et al. (2013), which explores the effect of international capital flow taxation on default and welfare in a deterministic two period large open economy; and Walsh (2015*a*) and Walsh (2015*b*), which consider default in a small open dynamic incomplete markets economy. In these latter two papers, the marginal cost of default depends on the level of wealth, so the propensity to default depends on business cycle fluctuations. We follow this notion here by introducing a macrovariable that governs the marginal cost of renegotiating debt (default), termed 'credit conditions'. This reflects changing motivations and incentives of debtors to make the necessary sacrifices to repay their obligations, as emphasised by Roch et al. (2016).

Ultimately the non-pecuniary default cost methodology and credit-conditions variable allows us to calibrate the model to realised average default rates. We believe that this approach has valid economic grounds and argue that credit-conditions can be adequately captured by an appropriate state variable in order to describe the relationship between loan delinquencies and the capital stock. Meanwhile the debtor firm takes the credit-conditions variable as given since creditors are capable of imposing institutional arrangements that are non-negotiable.

2.4 Supply Side

We denote a nominal variable with tilde as $\tilde{x}$ and real as x.

2.4.1 Wholesale Goods Producers

At each period t a measure 1 of firms, indexed by f, are born who live for two periods only. Each firm is capitalised (endowed) by their owners, the households, with nominal wealth $\tilde{e}_t^f$ (equity capital). In the first period they supplement their equity and income from physical capital sales with a loan from the banking system, $\tilde{\mu}_{t+1}^f$ which is composed of secured $\tilde{\mu}_{t+1}^{f,s}$ and unsecured $\tilde{\mu}_{t+1}^{f,u}$ debt. Secured debt is subject to a collateral constraint which limits the future amount of repayment by the expected future value of undepreciated capital stock. An unsecured debt is risky and can be defaulted upon. As all banks offer the same interest rates on the same type of debt, they are indifferent between which bank to obtain loans from and in fact borrow from both types of banks (systemically important large banks and not systemically important small banks). With these resources, the firms purchase new physical capital from capital producers, k_{t+1}^f at price P_t^K, to be used as an input to production at $t+1$.

At $t+1$, firms born at t, learn the productivity of their production technology (total factor prodcutivity, TFP). With probability $1-\theta_f$ firms have a relatively high efficiency of production $\bar{A}$ while with probability θ_f firms have a low productivity $\underline{A}_{t+1} < \bar{A}_{t+1}$. The distribution of productivities is independently and identically distributed and reflects the idiosyncratic risk of firms. Given their realised productivities, and the prevailing wage rate per unit of labour of $\tilde{W}_{t+1}$ and price for final output P_{t+1}^N, firms hire a quantity of labour, l_{t+1}^f. The total production is given by a constant returns to scale production function:

$$y_{t+1}^f = A_{t+1}^f (k_{t+1}^f)^\alpha (l_{t+1}^f)^{1-\alpha}. \tag{1}$$

In the second period firms capital depreciates at rate τ and remaining physical capital, $(1-\tau)k_t^f$, is transferred to new born firms. Given their net revenue from production, $P_{t+1}^N y_{t+1}^f - \tilde{w}_{t+1} l_{t+1}^f$, they evaluate how much of their unsecured debt $\tilde{\mu}_{t+1}^{f,u}(1+i_t^{f,u})$ to honour, deciding on a rate of repayment $1-\delta_{t+1}^f$, and how much profit to distribute to shareholders $\tilde{\pi}_{t+1}^f$. If they decide to default on δ_{t+1}^f % of their unsecured debt, firm management is required to exert an effort cost of $\frac{\tilde{\Omega}_{t+1}^f}{2}\Big(\delta_{t+1}^f \tilde{\mu}_{t+1}^{f,u}(1+i_t^{f,u})\Big)^2$. Thus firms trade-off the marginal value of additional dividend payments to shareholders vs the additional effort-cost of renegotiating debt.

Formally, the first period budget constraint of firm f is:

$$P_t^K k_{t+1}^f + \tilde{T}_t^f + \frac{a_\mu}{2}\left(\tilde{\mu}_{t+1}^{f,u} - \bar{\tilde{\mu}}^{f,u}\right)^2 + \frac{a_\mu}{2}\left(\tilde{\mu}_{t+1}^{f,s} - \bar{\tilde{\mu}}^{f,s}\right)^2 + \frac{a_K}{2}\left(\tilde{k}_{t+1}^f - \bar{\tilde{k}}^f\right)^2 = \tilde{\mu}_{t+1}^f + (1-\tau)P_t^K k_t^f + \tilde{e}_t^f \tag{2}$$

Where $\tilde{\mu}_{t+1}^f = \tilde{\mu}_{t+1}^{f,s} + \tilde{\mu}_{t+1}^{f,u}$. Firms are subject to the collateral constraint which limits the amount of secured debt that they can borrow from the bank:

$$\mathbb{E}(1+i_t^{f,s})\tilde{\mu}_{t+1}^{f,s} \leq coll(1-\tau)k_{t+1}^f \,\mathbb{E}\, P_{t+1}^K \tag{3}$$

where $coll$ is the margin of the collateral constraint. The second period budget constraint is:

$$\tilde{\pi}_{t+1}^f + (1-\delta_{t+1}^f)\tilde{\mu}_{t+1}^{f,u}(1+i_t^{f,u}) + \tilde{\mu}_{t+1}^{f,s}(1+i_t^{f,s}) + \tilde{w}_{t+1}l_{t+1}^f = P_{t+1}^N A_{t+1}^f (k_{t+1}^f)^\alpha (l_{t+1}^f)^{1-\alpha} - \frac{\tilde{\Omega}_{t+1}^f}{2}\left(\delta_{t+1}^f \tilde{\mu}_{t+1}^{f,u}(1+i_t^{f,u})\right)^2 + \tilde{p}_{t+1}^K k_{t+1}^f (1-\tau) \tag{4}$$

We obtain the real budget constraints by dividing the above by the domestically-priced final goods price level P_t. We define the real price of capital as $p_t^K = \frac{P_t^K}{P_t}$, and the real interest rate on secured and unsecured loans as $(1+r_{t+1}^{f,s}) = \frac{(1+i_t^{f,s})}{1+\pi_{t+1}}$ and $(1+r_{t+1}^{f,u}) = \frac{(1+i_t^{f,u})}{1+\pi_{t+1}}$.

$$p_t^K k_{t+1}^f + T_t^f + \frac{a_\mu}{2}\left(\mu_{t+1}^{f,u} - \bar{\mu}^{f,u}\right)^2 + \frac{a_\mu}{2}\left(\mu_{t+1}^{f,s} - \bar{\mu}^{f,s}\right)^2 + \frac{a_K}{2}\left(k_{t+1}^f - \bar{k}^f\right)^2 = \mu_{t+1}^f + (1-\tau)p_t^K k_t^f + e_t^f \tag{5}$$

$$\mathbb{E}(1+r_{t+1}^{f,s})\mu_{t+1}^{f,s} \leq coll(1-\tau)k_{t+1}^f \,\mathbb{E}\, p_{t+1}^K \tag{6}$$

$$\pi_{t+1}^f + (1-\delta_{t+1}^f)\mu_{t+1}^{f,u}(1+r_{t+1}^{f,u}) + \mu_{t+1}^{f,s}(1+r_{t+1}^{f,s}) + w_{t+1}l_{t+1}^f = p_{t+1}^N A_{t+1}^f (k_{t+1}^f)^\alpha (l_{t+1}^f)^{1-\alpha} - \frac{\Omega_{t+1}^f}{2}\left(\delta_{t+1}^f \mu_{t+1}^{f,u}(1+r_{t+1}^{f,u})\right)^2 + p_{t+1}^K k_{t+1}^f (1-\tau) \tag{7}$$

Given prices, $\left\{p_t^N, p_t^K, w_{t+1}, r_{t+1}^{f,s}, r_{t+1}^{f,u}\right\}$, endowments $\left\{e_t^f, k_t^f\right\}$ and technology $\left\{A_{t+1}^f\right\}$, firms choose $\left\{k_{t+1}^f, \mu_{t+1}^{f,u}, \mu_{t+1}^{f,s}, l_{t+1}^f, \delta_{t+1}^f\right\}$ to maximise

$$\max_{k_{t+1}^f, \mu_{t+1}^{f,u}, \mu_{t+1}^{f,s}, l_{t+1}^f, \delta_{t+1}^f} \mathbb{E}\, \beta^{sav} \Lambda_{t+1}^{sav} \left[\Pi_{t+1}^f\right] \tag{8}$$

Where Λ^{sav}_{t+1} is the marginal value of income for the Saver Household, and $\frac{\Omega^f_{t+1}}{2}\left(\delta^f_{t+1}\mu^{f,u}_{t+1}(1+r^{f,u}_{t+1})\right)^2$ is the pecuniary cost of default. For unlucky firms Ω^f_{t+1} is positive and bounded from above, for lucky firms it is infinity.

Ω^f_{t+1} varies with the aggregate debt, but individual firms do not internalize how their borrowing decisions affect the cost of default - pecuniary externality. For them Ω^f_{t+1} is a constant. In reality it evolves according to: $\Omega^f_t = \lambda^f \frac{\int \mu^{f,u}_{ss} df(1+r^{f,u}_{ss})(\delta^f_{ss})^{\gamma_1}}{K_{ss}p^K_{ss}} \frac{K_t p^K_t}{\int \mu^{f,u}_t df(1+r^{f,u}_t)(\delta^f_t)^{\gamma_1}}$, where λ^f is a steady state value for Ω^f_t.

Optimality requires:

with respect to k^f_{t+1}:

$$\mathbb{E}\left[\beta^{sav}\Lambda^{sav}_{t+1}p^N_{t+1}\alpha(k^f_{t+1})^{\alpha-1}A^f_{t+1}(l^f_{t+1})^{1-\alpha}+p^K_{t+1}(1-\tau)\right] = p^K_t\lambda^f_t(1+a_K(k^f_{t+1}-\bar{k}^f)) - \omega^f_t coll(1-\tau)\,\mathbb{E}\,p^K_{t+1}, \tag{9}$$

with respect to $\mu^{f,u}_{t+1}$:

$$\mathbb{E}\left[\beta^{sav}\Lambda^{sav}_{t+1}(1-\theta_f\delta^f_{t+1}+\theta_f\Omega^f_{t+1}(\delta^f_{t+1})^2(1+r^{f,u}_{t+1})\mu^{f,u}_{t+1})(1+r^{f,u}_{t+1})\right] = \lambda^f_t(1-a_\mu(\mu^{f,u}_{t+1}-\bar{\mu}^{f,u})), \tag{10}$$

with respect to $\mu^{f,s}_t$:

$$\mathbb{E}\left[\beta\Lambda^{sav}_{t+1}(1+r^{f,s}_{t+1})\right] = \lambda^f_t(1-a_\mu(\mu^{f,s}_{t+1}-\bar{\mu}^{f,s})) - \mathbb{E}\,\omega^f_t(1+r^{f,s}_{t+1}) \tag{11}$$

where λ^f_t is a Lagrange multiplier on the first period budget constraint. Λ^{sav}_t -marginal utility of saver households defined as $\Lambda^{sav}_t = u'(c^{sav,N}_t)$.

with respect to l^f_{t+1}

$$p^N_{t+1}(1-\alpha)(\bar{l}^f_{t+1})^{-\alpha}(k^f_{t+1})^\alpha\bar{A}^f_{t+1} = w_{t+1} \tag{12}$$

$$p^N_{t+1}(1-\alpha)(\underline{l}^f_{t+1})^{-\alpha}(k^f_{t+1})^\alpha\underline{A}^f_{t+1} = w_{t+1} \tag{13}$$

with respect to δ^f_{t+1}:

$$\Omega^f_{t+1} = \frac{1}{\delta^f_{t+1}\mu^{f,u}_{t+1}(1+r^{f,u}_{t+1})} \tag{14}$$

Where:

$$\mathbb{E}\, A^f_{t+1}(l^f_{t+1})^{1-\alpha} = \theta_f(\underline{A}^f_{t+1})(\underline{l}^f_{t+1})^{1-\alpha} + (1-\theta_f)\bar{A}^f_{t+1}(\bar{l}^f_{t+1})^{1-\alpha} \tag{15}$$

$$\mathbb{E}\,\Omega^f_{t+1}(\delta^f_{t+1})^2 = \mathbb{E}\,\frac{\theta_f}{\Omega^f_{t+1}(\mu^{f,u}_{t+1}(1+r^{f,u}_{t+1}))^2} \tag{16}$$

2.4.2 Intermediate goods producers

Intermediate Goods Producers are monopolistically competitive and produce a differentiated intermediate good using wholesale goods:

$$Y^{ret}_t(k) = Y^N_t(k) \tag{17}$$

Then an intermediate producer solves:

$$\min_{Y^{ret}_t(k)} \frac{P^N_t}{P_t}Y^{ret}_t(k) + \lambda^{ret}_t(Y^{ret}_t(k) - Y^N_t(k))$$

FOC:

$$\lambda^{ret}_t = \frac{P^N_t}{P_t} = p^N_t \tag{18}$$

Then the intermediate producer sets the price $p_t(k)$:

$$\max_{p_t(k)} \mathbb{E}_t \sum_{i=0}^{\infty}(\beta^{sav}\theta_{ps})^i\Lambda^{sav}_{t+i}\Big[\frac{p_t(k)}{P_{t+i}}c_{t+i}(k) - \lambda^{ret}_{t+i}c_{t+i}(k)\Big] \tag{19}$$

s.t. $Y^{ret}_t(k) = (\frac{p_t(k)}{P_t})^{-\theta_c}Y^{ret}_t$.
The solution to this problem is given by:

$$\mathbb{E}_t \sum_{i=0}^{\infty}(\beta^{sav}\theta_{ps})^i\Lambda^{sav}_{t+i}\Big[(1-\theta_c)\frac{p^*_t}{P_{t+i}} + \lambda^{ret}_{t+i}\theta_c\Big](\frac{p^*_t}{P_{t+i}})^{-\theta_c}\Big(\frac{1}{p^*_t}\Big)Y^{ret}_{t+i} = 0 \tag{20}$$

It can be shown that

$$(1+\pi_t)^{1-\theta_c} = (1-\theta_{ps})(1+\pi^*_t)^{1-\theta_c} + \theta_{ps} \tag{21}$$

where

$$Y^{ret} = Y^N_t/v^p_t \tag{22}$$

Price persistence v^p_t is defined as:

$$v_t^p = (1-\theta_{ps})(\frac{1+\pi_t}{1+\pi_t^*})^{\theta_c} + \theta_{ps}(1+\pi_t)^{\theta_c} v_{t-1}^p \tag{23}$$

2.4.3 Domestically-Priced Final Goods Producers

Domestically-Priced Final Goods Producers create a composite final good using as inputs goods purchased from intermediate goods producers that is then demanded by Saver Households, Borrower Households, the Government, and Capital Producers, and is given by:

$$Y_{t,y}^{ret} = \Big(\int_0^1 Y_{t,y}^{ret}(k)^{(\theta_c-1)/\theta_c} dk\Big)^{\frac{\theta_c}{(\theta_c-1)}} \tag{24}$$

It can be shown that the demand for the individual good k is given by:

$$Y_{t,y}^{ret}(k) = (\frac{p_t(k)}{P_t})^{-\theta_c} Y_{t,y}^{ret} \tag{25}$$

Where $Y_{t,y}^{ret}$ is the bundle of domestically-priced final goods consumed by each of the agents. Aggregating across agents we get:

$$\sum_y Y_{t,y}^{ret}(k) = Y_t^{ret}(k) = (\frac{p_t(k)}{P_t})^{-\theta_c} Y_t^{ret} \tag{26}$$

Where we assume that elasticities (θ_c) are identical across agents.

2.4.4 Foreign-Priced Final Goods

Foreign-Priced Final Goods are modelled as an endowment process Y^T. The price is set abroad as P_t^{*T} and traded domestically at a price of $P_t^T = Q_t P_t^{*T}$. Households can consume domestically, import or export it abroad.

2.4.5 Capital Production Sector

Capital producers purchase undepreciated capital $(1-\tau)K_t = (1-\tau)\int k_t^f df$ at price P_t^K from young Wholesale Firms, and purchase consumption goods i_t from the Final Goods market. Capital Producers combine both components into producing new capital $K_{t+1} = \int k_{t+1}^f d$, using the following production function:

$$K_{t+1} = (1-\tau)K_t + i_t\Big(1 - \frac{\varkappa}{2}\Big(\frac{\epsilon_t^K i_t}{i_{t-1}} - 1\Big)^2\Big) \tag{27}$$

Each capital producer, therefore, maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^{sav})^t \Lambda_t^{sav} \left[p_t^K (K_{t+1} - (1-\tau)K_t) - i_t \right] \tag{28}$$

This yields the following capital price equation:

$$\Lambda_t^{sav} = \begin{array}{c} \Lambda_t^{sav} p_t^K \left(1 - \frac{\varkappa}{2}\left(\frac{\varepsilon_t^K i_t}{i_{t-1}} - 1\right)^2 - \Lambda_t^{sav} \varkappa \left(\frac{\varepsilon_t^K i_t}{i_{t-1}} - 1\right) \frac{\varepsilon_t^K i_t}{i_{t-1}}\right) \\ + E_t \beta^{sav} \Lambda_{t+1}^{sav} \left[p_{t+1}^K \varkappa \left(\frac{\varepsilon_{t+1}^K i_{t+1}}{i_t} - 1\right) \left(\frac{\varepsilon_{t+1}^K i_{t+1}}{i_t}\right)^2 \right] \end{array}. \tag{29}$$

where $p_t^K = \frac{P_t^K}{P_t}$ is the real price of capital.

2.4.6 Copper Sector

The copper sector is modelled as in Hamann et al. (2016). A representative copper-extracting firm makes a decision of an copper extraction. At the beginning of a period t, the economy has res_t units of copper reserves and discovers a further $disc_t$ units. The copper-extracting firm then sells ext_t units in the competitive international copper market at foreign currency price $P_t^{o,*}$ which converts to a domestic currency price of P_t^o. The real domestic price is defined as $p_t^o = Qp_t^{o,*}$. The domestic currency cost of extracting ext_t is $\tilde{cost}(res_t, ext_t)$ while $cost(res_t, ext_t)$ is the real cost of extracting ext_t units of copper.

Profits (in nominal terms) and reserves of the firm are as follows:

$$\tilde{\Pi}_t^o = P_t^o ext_t - \tilde{cost}(res, ext), \tag{30}$$

while the resource constraint is

$$res_{t+1} + ext_t = res_t + disc_t. \tag{31}$$

Copper prices and discoveries follow AR(1) processes and we assume that $\tilde{cost}(res_t, ext_t) = P_t \frac{\kappa}{2} \frac{ext_t^2}{1+res_t}$.

Profits in real terms is given by:

$$\Pi_t^{ext} = p_t^o ext_t - cost(res_t, ext_t) \tag{32}$$

A representative firm solves then:

$$max_{ext_t, res_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left[(\beta^{sav})^t \Lambda_t^{sav} \Pi_t^o \right] \tag{33}$$

Optimality requires:

$$\Lambda_{t+1}^{sav}\beta^{sav}p_{t+1}^{ext} - \Lambda_t^{sav}p_t^{ext} + \Lambda_t^{sav}\frac{\kappa ext_t}{1+res_t} = \frac{\kappa\beta^{sav}\Lambda_{t+1}^{sav}}{2}\frac{(2ext_{t+1}(1+res_{t+1}) - ext_{t+1}^2)}{(1+res_{t+1})^2} \quad (34)$$

where p_t^o is the real copper price in terms of domestically-priced final goods. Copper prices and copper discoveries follow AR(1) processes.

2.5 Financial Sector: Banks

The main distinctive feature of the Chile banking sector is its pronounced heterogeneity in market share. It can be viewed as having a two-level structure: the major part is concentrated within several large credit institutions while the other several thousands of banks together have only a small market share. We explicitly model this feature by introducing two types of two-period lived banks in our model: Systemically Important (Big) banks and Small banks

The Small banking sector consists of banks who take a relatively small share of the market. All banks are subject to capital requirements introduced by financial regulator. However, systemically important banks are subject to higher capital requirements. This is consistent with Basel. For example, while capital requirement for a small bank stays at 9% it is 11.5% for a systemically important bank.

All banks of any category are identical ex ante, because risky firms are identical. Ex post those banks that lent to unlucky firms will suffer from partial default on the loans. We call these banks 'unlucky' banks. 'Lucky' banks will suffer no default on loans that they lent. It should be noticed that systemically important banks will suffer less, because they invest less in risky firms ex ante.

Systemically important banks (big) lend to a pool of risky firms $\int \mu_t^{big,f} df$ so that ex post they are subject only to aggregate risk. In contrast, small banks $(small)$ can lend to only a one firm and are exposed to individual firm risk (both idiosyncratic and aggregate).

2.5.1 Systemically important (big) banks

New-born systemically important large banks are capitalised with equity of $\tilde{e}_t^{\gamma,big}$. They accept deposits from households of $\int \tilde{d}_{t+1}^{big,sav} d(sav)$ and extend loans to borrower households and each wholesale firm and trade government bonds. The total value of the loan portfolio to firms is $\int \tilde{\mu}_{t+1}^{big,f} df$ and to borrower households $\int \mu_{t+1}^{big,borr} d(borr)$. Saver Households are shareholders of the banks and invest $\tilde{e}_t^{\gamma,big}$. In the second period banks obtain income from loans extended firms, borrower households and banks.

The first period budget constraint of a systemically important bank is given by

$$\int \mu_{t+1}^{big,f} df + b_{t+1}^{big} + \int \mu_{t+1}^{big,borr} d(borr) = \int d_{t+1}^{big,sav} d(sav) + e_t^{\gamma,big} - \frac{a_\mu}{2}\left(\mu_{t+1}^{big,f,u} - \bar{\mu}^{big,f,u}\right)^2$$
$$- \frac{a_\mu}{2}\left(\mu_{t+1}^{big,f,s} - \bar{\mu}^{big,f,s}\right)^2 - \frac{a_d}{2}\left(d_{t+1}^{big,sav} - \bar{d}^{big,sav}\right)^2 - \frac{a_b}{2}\left(b_{t+1}^{big} - \bar{b}^{big}\right)^2 \tag{35}$$

Capital adequacy concerns require banks to hold a proportion of the risk-weighted assets $\left(k_t^{big,\gamma}\right)$ over the requirements $\bar{k}^{big}$. Failure to maintain the required amount results in a quadratic pecuniary penalty.[2]

The capital adequacy ratio is defined as the ratio of bank capital to risk weighted assets net of reserves ($rwa_t^{big,\gamma}$), for a systemically important bank:

$$k_t^{big,\gamma} = \frac{e_t^{\gamma,big}}{rwa_t^{big,\gamma}} =$$
$$= \frac{e_t^{\gamma,big}}{\left(\int r\bar{w}_t^{big,f,u} \mu_{t+1}^{big,f,u} df + \int r\bar{w}_t^{big,f,s} \mu_{t+1}^{big,f,s} df + \int r\bar{w}_t^{big,borr} \mu_{t+1}^{big,borr} d(borr) + r\bar{w}_t^{big,b} b_{t+1}^{big}\right)} \tag{36}$$

Where $r\bar{w}_t^{big,f,u}$, for example, is the risk weight assigned to unsecured assets of a big bank $rw_t^{big,f,u} = r\bar{w}^{big,f,u} + .4 * \mathbb{E}\delta_{t+1}^f$. The profit function of a systemically important bank in nominal terms is given by:

Big banks then choose how much of secured and unsecured debt to lend out to firms, banks and households:

$$\begin{aligned} \Pi_{t+1}^{\gamma,big} = {} & [\theta_f(1 + r_{t+1}^{f,u}) \int (1 - \delta_{t+1}^f)\mu_{t+1}^{big,f,u} df + (1 - \theta_f)(1 + r_{t+1}^{big,f,u}) \int \mu_{t+1}^{big,f,u} df + \\ & + (1 + r_{t+1}^{f,s}) \int \mu_{t+1}^{big,f,s} df + (1 + r_{t+1}^{h}) \int \mu_{t+1}^{big,borr} d(borr)] + \\ & + (1 + r_{t+1}^{b}) b_{t+1}^{big} - [(1 + r_{t+1}^{d}) \int d_{t+1}^{big,sav} d(sav)] \end{aligned} \tag{37}$$

Given $\left\{\delta_{t+1}^f, r_{t+1}^{f,u}, r_{t+1}^{f,s}, r_{t+1}^{h}, r_{t+1}^{b}, r_{t+1}^{d}\right\}$, banks choose $\left\{\mu_{t+1}^{big,f,u}, \mu_{t+1}^{big,f,s}, \mu_{t+1}^{big,borr}, b_{t+1}^{big}, d_{t+1}^{big,sav}\right\}$ to maximize the following objective function:

[2]Penalising the banks for being below the requirement reflects the inefficiency of the bank's intermediation process.

$$\mathbb{E}_t\,\beta^{big}\frac{(\hat{\Pi}^{\gamma,big}_{t+1})^{1-\varsigma_{big}}}{1-\varsigma_{big}} - a_{cap}0.5[k^{\gamma,big}_t - \bar{k}^{big}]^2 \tag{38}$$

Optimality requires, with respect to unsecured loans (integrating over loans):

$$\begin{aligned}
&\mathbb{E}\,\frac{\beta^{big}}{(\hat{\Pi}^{big}_{t+1})^{\varsigma_{big}}}\Big(\Big\{\theta_f(1-\delta^f_{t+1}) + (1-\theta_f)\Big\}(1+r^{f,u}_{t+1})\Big)+ \\
&+ a_{cap}[k^{\gamma,big}_t - \bar{k}^{big}]\frac{rw^{big,f}e^{\gamma,big}_t}{(rwa^{big,\gamma}_t)^2} = \lambda^{big}_t(1 + a_\mu\int(\mu^{big,f,u}_{t+1} - \bar{\mu}^{big,f,u})df)
\end{aligned} \tag{39}$$

with respect to secured loans (integrating over loans):

$$\begin{aligned}
&\mathbb{E}\,\frac{\beta^{big}}{(\hat{\Pi}^{sav}_{t+1})^{\varsigma_{big}}}\Big(1+r^{f,s}_{t+1}\Big) + a_{cap}[k^{\gamma,big}_t - \bar{k}^{big}]\frac{rw^{big,f}e^{\gamma,big}_t}{(rwa^{big,\gamma}_t)^2} \\
&= \lambda^{big}_t(1 + a_\mu\int(\mu^{big,f,s}_{t+1} - \bar{\mu}^{big,f,s})df)
\end{aligned} \tag{40}$$

with respect to government bonds:

$$\mathbb{E}\,\frac{\beta^{big}}{(\hat{\Pi}^{big}_{t+1})^{\varsigma_{big}}}\Big((1+r^b_{t+1})\Big) = \lambda^{big}_t(1 + a_b(b^{big}_{t+1} - \bar{b}^{big}) \tag{41}$$

with respect to deposits:

$$\mathbb{E}\,\frac{\beta^{big}}{(\hat{\Pi}^{big}_{t+1})^{\varsigma_{big}}}\Big((1+r^d_{t+1})\Big) = \lambda^{big}_t(1 - a_d\int(d^{big,sav}_{t+1} - \bar{d}^{big,sav})d(sav)) \tag{42}$$

with respect to loans to borrower households:

$$\mathbb{E}\,\frac{\beta^{big}}{(\hat{\Pi}^{big}_{t+1})^{\varsigma_{big}}}(1+r^h_{t+1}) = \lambda^{big}_t(1 + a_\mu\int(\mu^{big,borr}_{t+1} - \bar{\mu}^{big,borr})d(borr)) \tag{43}$$

2.5.2 Small banks

Small banks are not considered systemically important. As they are small, their loan portfolio is less diversified than the large banks. This means that small banks lend only to one type of firm and, ex-post, the bank's portfolio experiences the idiosyncratic risk emanating from firms. The first period budget constraint of a small bank in real terms:

$$\mu_{t+1}^{small,f} + \mu_{t+1}^{small,borr} d(borr) = d_{t+1}^{small,sav} + e_t^{\gamma,small} - \frac{a_\mu}{2}\Big(\mu_{t+1}^{small,f,s} - \bar{\mu}^{small,f,s}\Big)^2 - \frac{a_\mu}{2}\Big(\mu_{t+1}^{small,f,u} - \bar{\mu}^{small,f,u}\Big)^2 - \frac{a_d}{2}\Big(d_{t+1}^{small,sav} - \bar{d}^{small,sav}\Big)^2 \tag{44}$$

In real terms the second period budget constraint is given by:

$$\Pi_{t+1}^{\gamma,small} = [(1+r_{t+1}^{u,f})(1-\delta_{t+1}^f)\mu_{t+1}^{small,f,u} + (1+r_{t+1}^{f,s})\mu_{t+1}^{small,f,s} + (1+r_{t+1}^h)\mu_{t+1}^{small,borr}] + - (1+r_{t+1}^d)d_{t+1}^{small,borr} \tag{45}$$

For a small bank capital adequacy ratio looks like:

$$k_t^{\gamma,small} = \frac{e_t^{\gamma,small}}{rwa_t^{\gamma,small}} = = \frac{e_t^{\gamma,small}}{rw_t^{small,f,u}\mu_{t+1}^{small,f,u} + rw_t^{small,f,s}\mu_{t+1}^{small,f,s} + rw_t^{small,borr}\mu_{t+1}^{small,borr}} \tag{46}$$

where $rw^{small,f,u}$, for example, is the risk weight assigned to unsecured assets of a small bank: $rw_t^{small,f,u} = r\bar{w}^{small,f,u} + .4 * \mathbb{E}\delta_{t+1}^f$.

Given $\Big\{\delta_{t+1}^f, r_{t+1}^{f,u}, r_{t+1}^{f,s}, r_{t+1}^h, r_{t+1}^d\Big\}$, banks choose $\Big\{\mu_{t+1}^{small,f,u}, \mu_{t+1}^{small,f,s}, \mu_{t+1}^{small,borr}, \mu_{t+1}^{small,\gamma}, d_{t+1}^{small,sav}\Big\}$ to maximize the following objective function:

$$\mathbb{E}_t\, \beta^{sav}\Lambda_{t+1}^{sav}\Big(\theta_f \frac{(\underline{\Pi}_{t+1}^{\gamma,small})^{1-\varsigma_{small}}}{1-\varsigma_{small}} + (1-\theta_f)\frac{(\bar{\Pi}_{t+1}^{\gamma,small})^{1-\varsigma_{small}}}{1-\varsigma_{small}}\Big) - \Lambda_t^{sav} 0.5 a_{cap}[k_t^{\gamma,small} - \bar{k}^{small}]^2 \tag{47}$$

Optimality requires, with respect to unsecured loans:

$$\mathbb{E}\,\beta^{sav}\Lambda_{t+1}^{sav}\Big[\frac{\theta_f}{(\underline{\Pi}_{t+1}^{\gamma,small})^{\varsigma_{small}}}\Big((1-\delta_{t+1}^f)(1+r_{t+1}^{f,u})\Big) + \frac{1-\theta_f}{(\bar{\Pi}_{t+1}^{\gamma,small})^{\varsigma_{small}}}\Big((1+r_{t+1}^{f,u})\Big)\Big] + + \Lambda_t^{sav} a_{cap}[k_t^{\gamma,small} - \bar{k}^{small}]\frac{rw_t^{small} e_t^{\gamma,small}}{(rwa_{t+1}^{\gamma,small})^2} = \lambda_t^{small}(1 + a_\mu(\mu_{t+1}^{small,f,u} - \bar{\mu}^{small,f,u})) \tag{48}$$

with respect to secured loans:

$$\mathbb{E}\,\beta^{sav}\Lambda^{sav}_{t+1}\Big[\frac{\theta_f}{(\underline{\Pi}^{\gamma,small}_{t+1})^{\varsigma_{small}}}\Big((1+r^{f,s}_{t+1})\Big)+\frac{1-\theta_f}{(\bar{\Pi}^{\gamma,small}_{t+1})^{\varsigma_{small}}}\Big((1+r^{f,s}_{t+1})\Big)\Big]+$$
$$+\Lambda^{sav}_t a_{cap}[k^{\gamma,small}_t-\bar{k}^{small}]\frac{rw^{small}_t e^{\gamma,small}_t}{(rwa^{\gamma,small}_{t+1})^2}=\lambda^{small}_t(1+a_\mu(\mu^{small,f,s}_{t+1}-\bar{\mu}^{small,f,s})) \tag{49}$$

with respect to deposits:

$$\mathbb{E}\,\beta^{sav}\Lambda^{sav}_{t+1}\Big[\frac{\theta_f}{(\underline{\Pi}^{\gamma,small}_{t+1})^{\varsigma_{small}}}\Big((1+r^{d}_{t+1})\Big)+\frac{1-\theta_f}{(\bar{\Pi}^{\gamma,small}_{t+1})^{\varsigma_{small}}}\Big((1+r^{d}_{t+1})\Big)\Big]=$$
$$=\lambda^{small}_t(1-a_d(d^{small,sav}_{t+1}-\bar{d}^{small,sav})) \tag{50}$$

with respect to borrower households:

$$\mathbb{E}\,\beta^{sav}\Lambda^{sav}_{t+1}\Big[\frac{\theta_f}{(\underline{\Pi}^{\gamma,small}_{t+1})^{\varsigma_{small}}}\Big(1+r^{h}_{t+1}\Big)+\frac{1-\theta_f}{(\bar{\Pi}^{\gamma,small}_{t+1})^{\varsigma_{small}}}\Big(1+r^{h}_{t+1}\Big)\Big]=$$
$$=\lambda^{small}_t(1-a_\mu(\mu^{small,borr}_{t+1}-\bar{\mu}^{small,borr})) \tag{51}$$

2.6 Demand Side: Households

2.6.1 Borrowers

Borrower Households choose how much labor to supply, how much to consume and how much to borrow from both types of banks. Borrower Households do not own firms and banks, but they receive transfers from the Government.

We assume that the consumption goods basket for the Borrower and Saver household is a Cobb-Douglas compound of the two final goods as follows:

$$c^{borr}_t=(c^{borr,N}_t)^{\varphi}(c^{borr,T}_t)^{1-\varphi} \tag{52}$$

Budget Constraint of a Household:

$$p^T_t c^{borr,T}_t+c^{borr,N}_t\leq\mu^{borr}_{t+1}-(1+r^h_t)\mu^{borr}_t+w_t l^{borr}_t+Tr_t \tag{53}$$

Household then chooses consumption of final goods, loans and labor supply by maximizing the following objective function s.t. the BC:

$$\sum_{t=0}^{\infty}(\beta^{borr})^t[\frac{(c_t^{borr})^{1-\sigma}}{1-\sigma} - \gamma^{borr}\frac{(l_t^{borr})^2}{2}]$$

F.O.C. for μ_{t+1}^{borr}:

$$\lambda_t^{borr} = \mathbb{E}_t \beta^{borr}(1+r_{t+1}^h)\lambda_{t+1}^{borr} \tag{54}$$

F.O.C. for $c_t^{borr,N}$:

$$\frac{(c_t^{borr,N})^{\phi-1}(c_t^{borr,T})^{1-\phi}}{(c_t^{borr})^{\sigma}} = \frac{\lambda_t^{borr}}{\phi} \tag{55}$$

F.O.C. for $c_t^{borr,T}$:

$$\frac{(c_t^{borr,N})^{\phi}(c_t^{borr,T})^{-\phi}}{c_t^{\sigma}} = \frac{\lambda_t^{borr}p_t^T}{1-\phi} \tag{56}$$

F.O.C. for l_t^{borr}:

$$\gamma^{borr}l_t^{borr} = \lambda_t^{borr}w_t \tag{57}$$

where λ_t^{borr} is a Lagrange multiplier.

2.6.2 Savers

Saver Households choose how much labor to supply and how much to consume. They save at both banks. Saver Households receive their profits from lucky and unlucky wholesale firms, intermediate producers, banks, capital producers and a proportion of capital from old firms at that date, $(1-\tau)K_{t-1}$ which they sell to capital producers for price P_t^K.

We assume that the consumption goods basket is given by:

$$c_t^{sav} = (c_t^{sav,N})^{\varphi}(c_t^{sav,T})^{1-\varphi} \tag{58}$$

Budget Constraint of a Saver Household:

$$
\begin{aligned}
& d_{t+1}^{sav} + p_t^T c_t^{sav,T} + c_t^{sav,N} + e_t^f + (1-\tau)p_t^K K_t + e_t^{\gamma,small} + e_t^{\gamma,big} \\
& \leq p_t^T Y_t^T + (1+r_t^d)d_t^{sav} + w_t l_t^{sav} + (1-\theta)\int \bar{\Pi}_t^f df + \theta \int \underline{\Pi}_t^f df + \\
& + (1-\theta)\int \bar{\Pi}_t^{\gamma,small} d\gamma + \theta \int \underline{\Pi}_t^{\gamma,small} d\gamma + \int \Pi_t^{\gamma,big} d\gamma + \Pi_t^{cap} + \Pi_t^{ret} - T_t - \\
& - 0.5adj_e(e_{ss}^{\gamma,small} - e_{ss}^{\gamma,small})^2 - 0.5adj_e(e_{ss}^{\gamma,big} - e_{ss}^{\gamma,big})^2 - \\
& - 0.5adj_e(e_t^f + (1-\tau)p_t^K K_t - (e_{ss}^f + (1-\tau)p_{ss}^K K_{ss}))^2
\end{aligned} \tag{59}
$$

Deposits are not subject to default risk due to Deposit Insurance. In period t, banks are obliged to repay the amount of $\left(1+r_t^d\right) d_t^{sav}$. Saver Households transfer equity to firms and small banks $e_t^{\gamma,small} + e_t^{\gamma,big} + e_t^f$. Saver Household then chooses consumption of final goods, deposits, equity invested and labor supply by maximizing the following objective function s.t. the BC:

$$
\sum_{t=0}^{\infty} (\beta^{sav})^t [\frac{(c_t^{sav})^{1-\sigma}}{1-\sigma} - \gamma^{sav} \frac{(l_t^{sav})^2}{2}]
$$

F.O.C. for μ_{t+1}^{sav}:

$$
\lambda_t^{sav} = \mathbb{E}_t \, \beta^{sav}(1+r_{t+1}^d)\lambda_{t+1}^{sav} \tag{60}
$$

F.O.C. for $c_t^{sav,N}$:

$$
\frac{(c_t^{sav,N})^{\phi-1}(c_t^{sav,T})^{1-\phi}}{(c_t^{sav})^{\sigma}} = \frac{\lambda_t^{sav}}{\phi} \tag{61}
$$

F.O.C. for $c_t^{sav,T}$:

$$
\frac{(c_t^{sav,N})^{\phi}(c_t^{sav,T})^{-\phi}}{c_t^{\sigma}} = \frac{\lambda_t^{sav} p_t^T}{1-\phi} \tag{62}
$$

F.O.C. for l_t^{sav}:

$$
\gamma^{sav} l_t^{sav} = \lambda_t^{sav} w_t \tag{63}
$$

F.O.C. for $e_t^{\gamma,small}$:

$$
e_t^{\gamma,small}(1 + adj_e(e_t^{\gamma,small} - e_{ss}^{\gamma,small})) = \frac{\lambda_{t+1}^{sav}\beta^{sav}}{\lambda_t^{sav}}(\theta_f \underline{\Pi}_t^{\gamma,small} + (1-\theta_f)\bar{\Pi}_t^{\gamma,small}) \tag{64}
$$

F.O.C. for $e_t^{\gamma,big}$:

$$e_t^{\gamma,big}(1+adj_e(e_t^{\gamma,big}-e_{ss}^{\gamma,big}))=\frac{\lambda_{t+1}^{sav}\beta^{sav}}{\lambda_t^{sav}}(\Pi_t^{\gamma,big}) \tag{65}$$

F.O.C. for e_t^f:

$$\begin{aligned}&(e_t^f+(1-\tau)p_t^K K_t)(1+adj_e(e_t^f+(1-\tau)p_t^K K_t-(e_{ss}^f+(1-\tau)p_{ss}^K K_{ss})))=\\&=\frac{\lambda_{t+1}^{sav}\beta^{sav}}{\lambda_t^{sav}}((1-\tau)p_{t+1}^K K_t+\underline{\Pi}_{t+1}^f\theta_f+\bar{\Pi}_{t+1}^f(1-\theta_f))\end{aligned} \tag{66}$$

where λ_t^{sav} is a Lagrange multiplier.

2.7 Government

In this section we introduce the Government. The Government consumes domestically-produced final goods, collects lump-sum taxes from Households and trades bonds on the interbank market. The Government owns the copper production sector. The Government budjet constraint (in real terms):

$$G_t+Tr_t+B_{t-1}^g\frac{(1+i_{t-1}^{ib})}{1+\pi_t}\leq B_t^g+\Pi_t^o+cost_t(res,ext)+T_t+\int T_t^f df \tag{67}$$

We assume that in the steady state $B_{ss}^g=0$.

2.7.1 The Central Bank

The Central Bank controls the interest rate i_t^{ib} according to the following rule:

$$\frac{1+i_t^{ib}}{1+i_{ss}^{ib}}=\Big(\frac{1+i_{t-1}^{ib}}{1+i_{ss}^{ib}}\Big)^{r_R}\Big(\frac{1+\pi_t}{1+\pi_{ss}}\Big)^{(1+r_\pi)(1-r_R)}\Big(\frac{Y_t}{Y_{ss}}\Big)^{r_Y(1-r_R)}\epsilon_t^R \tag{68}$$

where, r_π and r_y measure the sensitivity of the policy interest rate to deviations of inflation and aggregate output from the steady state. Monetary policy shocks ϵ_t^R are assumed to be i.i.d..

2.8 The CPI

We define the relative weight of foreign-priced final goods in the basket as $\omega_t^T=c_t^T/(c_t^T+c_t^N)$. Then the real cpi index is given by $rcpi_t=p_t^T\omega_t^T+(1-\omega_t^T)$. The real wage in terms of the CPI: $w_t^{cpi}=w_t/rcpi_t$.

The CPI inflation the is $\pi_t^{cpi}=(1+\pi_t)(rcpi_t)/(rcpi_{t-1})$ and the real deposit rate in terms of the CPI: $1+\rho_t^{cpi}=(1+\rho_t)(1+\pi_t)/\pi_t^{cpi}$.

2.9 Exchange Rate and Foreign-Priced Final Goods Prices

Households may purchase foreign-priced final goods domestically, or important them abroad. They face the same real price p_T in either case. However, as the goods are perfect substitutes, the domestic price of foreign-priced final goods is $p_T = Q_t p_T^*$. As the domestic price of copper is fixed at a steady state level of $\bar{p}^o$ which means that the real exchange rate is given by $Q_t = \frac{\bar{p}^o}{p_t^{o*}}$. Note that Q_t is the real exchange rate which, absent trade frictions, should be 1. Here we are assuming a degree of market segmentation which allows the real exchange rate to fluctuate inversely with the foreign price of copper.

2.10 Market clearing conditions

Final goods sector:

$$
\begin{aligned}
&Y_t^{ret} = c_t^{borr,N} + c_t^{sav,N} + G_t + i_t + \frac{a_K}{2}\left(k_{t+1}^f - \bar{k}^f\right)^2 \\
&+ \frac{a_d}{2}\left(d_{t+1}^{small,sav} - \bar{d}^{small,sav}\right)^2 + \frac{a_d}{2}\left(d_{t+1}^{big,sav} - \bar{d}^{big,sav}\right)^2 \\
&+ \frac{a_\mu}{2}\left(\mu_{t+1}^{big,f,u} - \bar{\mu}^{big,f,u}\right)^2 + \frac{a_\mu}{2}\left(\mu_{t+1}^{small,f,u} - \bar{\mu}^{small,f,u}\right)^2 \\
&+ \frac{a_\mu}{2}\left(\mu_{t+1}^{big,f,s} - \bar{\mu}^{big,f,s}\right)^2 + \frac{a_\mu}{2}\left(\mu_{t+1}^{small,f,s} - \bar{\mu}^{small,f,s}\right)^2 \\
&+ \frac{a_\mu}{2}\left(\mu_{t+1}^{big,borr} - \bar{\mu}^{big,borr}\right)^2 + \frac{a_\mu}{2}\left(\mu_{t+1}^{small,borr} - \bar{\mu}^{small,borr}\right)^2 \\
&+ \frac{a_\mu}{2}\left(\mu_{t+1}^{big,\gamma} - \bar{\mu}^{big,\gamma}\right)^2 + \frac{a_\mu}{2}\left(\mu_{t+1}^{small,\gamma} - \bar{\mu}^{small,\gamma}\right)^2 + \\
&+ 0.5adj_e(e_{ss}^{\gamma,small} - e_{ss}^{\gamma,small})^2 + 0.5adj_e(e_{ss}^{\gamma,big} - e_{ss}^{\gamma,big})^2 + \\
&+ 0.5adj_e(e_t^f + (1-\tau)p_t^K K_t - (e_{ss}^f + (1-\tau)p_{ss}^K K_{ss}))^2 + \frac{\Omega_{t+1}^f}{2}\left(\delta_{t+1}^f \mu_{t+1}^{f,u}(1 + r_{t+1}^{f,u})\right)^2
\end{aligned}
\tag{69}
$$

Labor market clears:

$$\int l_t^{sav} d(sav) + \int l_t^{borr} d(borr) = \theta_f \int \underline{l}_t^f df + (1-\theta_f) \int \bar{l}_t^f df \tag{70}$$

Deposit market:

$$\int_0^{\bar{\kappa}} d_t^{big} d\gamma + \int_{\bar{\kappa}}^1 d_t^{small} d\gamma = d_t^{sav} \tag{71}$$

Secured Loan market:

$$\mu_t^{f,s} = \int_0^{\bar{\kappa}} \mu_t^{big,f,s} d\gamma + \int_{\bar{\kappa}}^1 \mu_t^{small,f,s} d\gamma \tag{72}$$

Unsecured Loan market:

$$\mu_t^{f,u} = \int_0^{\bar{\kappa}} \mu_t^{big,f,u} d\gamma + \int_{\bar{\kappa}}^1 \mu_t^{small,f,u} d\gamma \tag{73}$$

Loans to Households (Borrowers):

$$\mu_t^{borr} = \int_0^{\bar{\kappa}} \mu_t^{big,borr} d\gamma + \int_{\bar{\kappa}}^1 \mu_t^{small,borr} d\gamma \tag{74}$$

Wholesale market:

$$Y_t^N = \theta_f \int \underline{y}_t^f df + (1 - \theta_f) \int \bar{y}_t^f df \tag{75}$$

Define real GDP as:

$$Y_t = Y_t^{ret} + p_t^o ext_t + p_t^T Y_t^T \tag{76}$$

3 Calibration

We have calibrated our economy using the Chilean financial and real sector data at a quarterly frequency. From the data we can also estimate the interest rate on deposits over the medium term as 5,67%. The loss given default rate for the firms is taken to be 50%. Probability of default θ_f is estimated to be equal to 10% (proportion of firms that default). The ratio of TFPs in high and low states are given by 1.2. The probability that the prices are sticky the next period is set to $\theta^{ps} = 0.7$, elasticity of substitution between different virieties $\theta^c = 10$. Income share of capital is $\alpha = 0.35$. In this setting the endogenously determined unsecured loans interest rate has a value of 11.23%.

As for the copper price, price of foreign-priced final goods and copper discoveries we estimate an AR(1) process and find the steady state. The parameters for the banking sector such as the relative risk weights assigned to banking assets rw and capital requirement ratios k are calibrated in accordance with the CCB policies and rules. We calibrate an asset share of systemically important banks as 65%. Capital requirement for small banks is 9% and for the big banks is set at 11.5%, risk weights - 100%.

Parameter	Description	Value
δ^f	Loss given default	0.5
r^d	Deposit rate	0.567
θ^f	Probability of default	0.1
$\bar{A}$	TFP process for lucky firms	12
$\underline{A}$	TFP process for unlucky firms	10
τ	Capital depreciation rate	0.05
α	Income share for capital	0.35
$\frac{\mu^{big,f}}{\mu^f}$	Share of systemically (s) important banks (loans)	0.65
$rw^{big,f}$	Risk weight for big banks	1
$rw^{small,f}$	Risk weight for ns banks	1
$\bar{k}^{big}$	Capital requirement for s banks	0.115
$\bar{k}^{small}$	Capital requirement for ns banks	0.09
p^T	dollar price of foreign-priced final goods	1
p^o	dollar price of copper	1
ς_{big}	Risk aversion of large banks	1
ς_{small}	Risk aversion of small banks	0
σ	Risk aversion of households	1.5
r_R	Taylor rule coefficient (interest rate elasticity)	0.82
r_π	Taylor rule coefficient (inflation elasticity)	0.57
r_Y	Taylor rule coefficient (output elasticity)	0.12
ρ	all autoregressive coefficients	0.9
$\varkappa$	capital production adjustment cost	1.74
κ	copper sector parameter	2
$disc$	copper discoveries	1
γ_1	parameter in the default cost function	0.955
a_μ	adjustment costs	0.01

Table 1: Calibrated parameters

4 Theoretical moments

Table **??** presents selected theoretical moments following a negative shock to the price of copper ($p^{o,*}$), an increase in the price of foreign-priced final goods ($p^{T,*}$), an increase in the interest rate response in the Taylor rule (ϵ^R), an increase in the cost of capital production ($\varkappa$), a decrease in the discoveries of copper ($disc$), and a decrease in the TFP of all wholesale producers (A).

Table **??** presents moments from data (García-Cicco and Kirchner (2015)). First order autocorrelations of aggregate consumption and TBtoGDP in the model are compatible with actual statistics. Standard deviations are quite different, and this might be explained by 1) the way we define the variables in the model, 2) the number and the size of shocks we use.

	Standard Deviation	1st order autocorrelation	Corr GDP
$\Delta log(GDP)\%$	13.624	0.678	1.000
$\Delta log(Y_{low})\%$	13.891	0.646	0.998
$\Delta log(Y_{high})\%$	13.891	0.646	0.998
$\Delta log(L_{high})\%$	19.909	0.599	0.743
$\Delta log(L_{low})\%$	19.909	0.599	0.743
$\Delta log(K)\%$	24.461	0.993	0.400
$\Delta log(int)\%$	61.657	0.971	-0.049
$\Delta log(p^T)\%$	3.244	0.900	-0.017
$\Delta log(ext^o)\%$	20.789	0.332	-0.650
$\Delta log(c^{sav,T})\%$	23.321	0.894	0.383
$\Delta log(c^{borr,T})\%$	59.035	0.736	0.848
$\Delta log(c^{sav,N})\%$	23.011	0.894	0.386
$\Delta log(c^{borr,N})\%$	58.946	0.735	0.849
$\Delta log(TBtoY)\%$	416.088	0.723	-0.702
$\Delta log(C)\%$	24.020	0.610	0.991
$\Delta log(\mu^{big})\%$	38.343	0.957	0.062
$\Delta log(\mu^{small})\%$	132.487	0.958	-0.216
$\Delta log(int)\%$	61.657	0.971	-0.049
$\Delta log(\Pi^{big})\%$	418.490	0.963	0.154
$\Delta log\Pi^{f,high}\%$	32.139	0.304	0.776
$\Delta log(\Pi^{f,low})\%$	29.466	0.239	0.742
$\Delta log(\Pi^{small,high})\%$	1650.581	0.958	-0.028
$\Delta log(Q)\%$	2.294	0.900	-0.012
r^d	0.194	0.411	-0.876
$r^{f,u}$	0.382	0.791	-0.695
$r^{f,s}$	0.192	0.374	-0.843
r^h	0.198	0.467	-0.894
δ^f	2.999	0.875	-0.517
$\Delta log(w)\%$	47.998	0.589	0.922
π	0.238	0.715	0.685

Table 2: Selected moments from the model

	Standard Deviation	1st order autocorrelation
$\Delta log(GDP)\%$	1,02	0,25
$\Delta log(C)\%$	1,1	0.63
$\Delta log(int)\%$	3,75	0.4
$\Delta log(TBtoGDP)\%$	5,32	0.73
$\Delta log(w)\%$	0.62	0.4

Table 3: Selected moments from the data (García-Cicco and Kirchner (2015))

5 Impulse Responses

In this section we discuss he impact and channels of transmissions of commodity price and productivity shocks. This is, real type of shocks that feedback into the financial system and have a feedback to the real economy.

5.1 Copper price shock

A negative copper price temporary shock (($p^{o,*}$), figure (2)) affects the foreign price of copper and transmits to the exchange rate depreciation as the domestic price of copper (in peso terms) remains roughly stable. [3]. An increase in the relative price of foreign-priced final goods affects the household optimal consumption decisions so that he substitutes a relatively more expensive foreign-priced final goods to domestically-priced final goods. However, the negative income effect leads to a fall in consumption of both final goods and households postpone their consumption as the rate of inflation measured with a CPI goes up. An increase in savings (household deposits) reduces the real rate on deposits. Due to the negative income effect there is an overall decrease in the demand for domestically-priced final goods, increase in the labor supply and a subsequent fall in the production level. As the default rate increases, the interest rate on unsecured loans goes up what increases the demand for labor. Together these factors lead to a decline in output. Banks reduce lending meeting a lower demand for the capital financing. This reduces profits considerably and leads to financial instability.

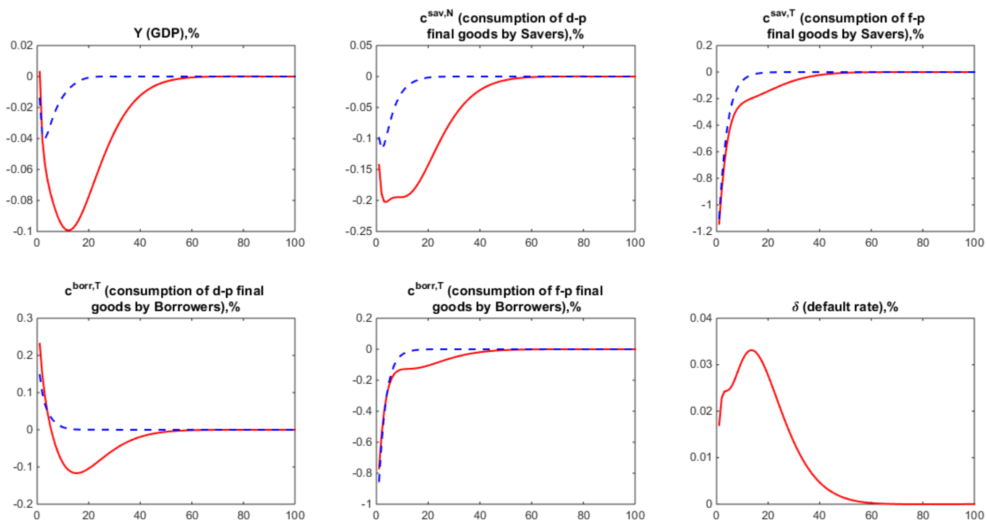

Figure 2: negative shock to the price of copper (deviations from steady state, months)

[3]We calibrate it using the data

5.2 TFP shock

A negative shock to TFP works in a similar fashion that the copper price one. However, the productivity decrease impacts the economic and financial system from within. It decreases all demands for different goods and hence consumption. The lower profits are translated into increased default rates that for several years. Additionally,it can be noticed that in this small open economy framework, internal shocks have a smaller impact as compared to external shocks that transmit through more channels.

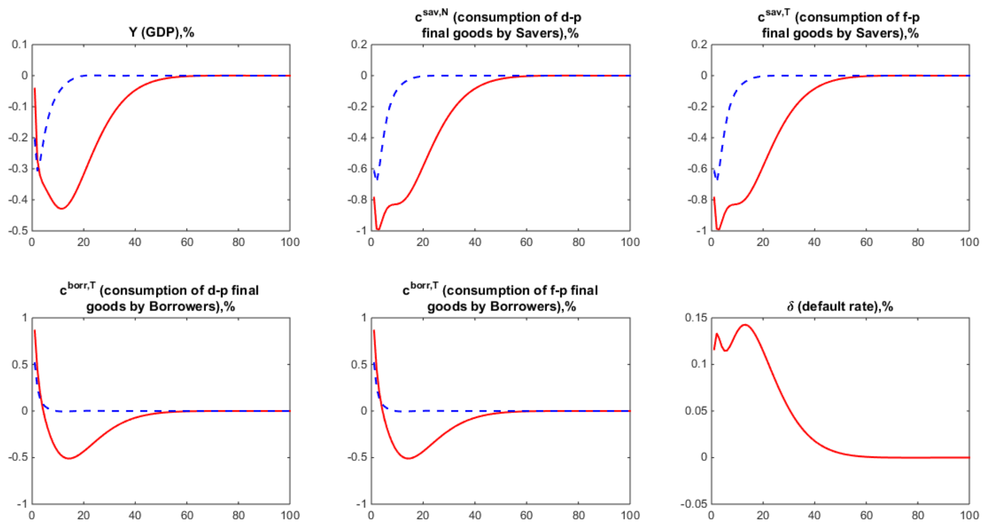

Figure 3: negative shock to TFP, deviations from steady state (months)

6 Conclusion

Given the development and implementation of macro and microprudential regulation, it is important for policymakers to assess their interaction with monetary policy. Our model exhibits several novel features, but emphasises the role that firm level heterogeneity, and corresponding cross-sectional default rates, plays as a source of financial instability in a DSGE model with a heterogenous banking sector. In particular, our modelling approach demonstrates that an adverse shock to copper price significantly has both real and financial effects that reinforce each other. In a stylized fashion, we capture the effects of copper prices on repayment rates of the real sector.Hence, default rates transmit to interest on unsecured borrowing and reduces investment. Our current research agenda includes studying to what extent regulation would help to stabilize the economy.

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