Interbank Network Disruptions and the Real Economy*

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Abstract

Interbank lending markets allow banks to resolve temporary imbalances and maintain smooth supply of credit to the nonfinancial sector; therefore, changes to the structure of these markets may have important macroeconomic repercussions. This paper examines the impact of shocks to the network of bank relationships (interbank network) on the real sector. First, by incorporating an interbank network into a dynamic general equilibrium model, I show that the aggregate interest rate on loans that banks provide to firms may *increase* or *decrease* in response to a shock that destroys bank relationships (network disruption), depending on the size of this shock and the initial distribution of bank relationships. This is in contrast with a notion that the aggregate interest rate on loans to the real sector unambiguously increases when the interbank market becomes less active. Second, I show that the central bank's policy can limit the extent to which the shocks that destroy bank relationships transmit onto the real sector. In particular, as the corridor between the discount window rate and the excess reserve rate decreases, the effect of a network disruption on the real sector becomes smaller.

Keywords: interbank network; liquidity; monetary policy
JEL classification: D85, E44, E52, G21

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1 Introduction

Interbank markets enhance the efficiency of the financial sector and allow banks to smoothly supply credit to the real sector. Traditionally, these markets have been excluded from macroeconomic and business-cycle analyses due to the perception that they are complete and frictionless.[1] However, the consequences of the distress in the financial sector observed in recent years calls the validity of this assumption into question. Furthermore, empirical studies of interbank markets document that banks do not typically utilize interbank markets to their full capacity, providing evidence for barriers that prevent banks from trading with each other.[2] Such barriers can arise as a result of asymmetric information, costly coordination, geographical/time constraints, or other frictions.[3] Because of these frictions, not all banks may have the same chance of finding a trading partner in the interbank lending market.

This paper is the first to consider bank-specific trading opportunities within a dynamic general equilibrium model. Even though this friction has been previously investigated in a static and/or partial equilibrium setting (for example, Allen and Gale (2000) and Freixas et al. (1998)), the existing models that analyze the banking sector within a dynamic general equilibrium environment abstract from bank-specific conditions in the interbank loan market, limiting our understanding of how changes in these conditions may affect the real economy. Given that central banks target the interest rate on short-term interbank loans, one crucial

[1]For example, Dynamic New Keynesian (DNK) model – the standard model used for monetary policy analysis – entirely abstracts from the interbank market, even though, in reality, the interest rate on loans in this market is the main monetary policy tool used by central banks.

[2]For empirical evidence that documents the sparse nature of interbank market participation see: Soramäki et al. (2007) and Bech and Atalay (2010) for evidence from the federal funds market; Iori et al. (2008) for Italian interbank market; Boss et al. (2004) for Austrian interbank market; Inaoka et al. (2004) for Japanese interbank market; Bräuning et al. (2012) and Craig and von Peter (2014) for German interbank network; and Vila et al. (2010) for unsecured overnight market in the United Kingdom.

[3]For example, Soramäki et al. (2007) find that participation in the interbank market fell following the attacks of September 11, 2001, due to the decrease in coordination which was, most likely, a result of operational problems. Calomiris and Carlson (2016) show that, during the National Banking era, banks in areas with more manufacturing firms maintained more network connections. Finally, a vast literature covers a surge in expected counterparty risk following the failure of Lehman Brothers. Some examples are Nier et al. (2007), Gupta et al. (2013), Blasques et al. (2016), Beltran et al. (2015).

aspect is how this transmission depends on monetary policy in an economy that features *both* heterogeneous and time-varying trading opportunities in the interbank lending market.

Because banks face liquidity shocks, which arise due to desynchronized revenues and outlays, they rely on the interbank loan market at times of cash shortages. Even though loans from the central bank are also available, they are typically more costly than interbank loans; therefore, banks attempt to borrow in the interbank market before relying on last-resort loans from the monetary authority. Depending on the distribution of interbank trading opportunities, some banks may be more successful in getting interbank loans than others, which gives rise to heterogeneous liquidity funding costs. A bank with a higher expected cost of financing liquidity does two things *ex ante*: first, it charges a higher interest rate on loans supplied to the real sector; and second, it chooses to hold a greater share of its assets in cash. All else equal, both actions result in a decline of a bank's lending to the real economy. What happens in a general equilibrium, however, depends on how the bank's trading opportunities compare to the trading opportunities of other banks at a given point in time. In this paper, I provide a framework that qualifies these general equilibrium effects.

More specifically, I incorporate a network of bank relationships into the Bianchi and Bigio (2014) framework. I define a relationship as a potential bilateral trading opportunity in the interbank loan market. While Bianchi and Bigio (2014) bring insights from the liquidity management literature into a dynamic macro model and propose a novel mechanism for monetary policy transmission, they implicitly assume that all banks are connected to each other (i.e. the interbank network is complete). By relaxing this assumption, I can address the following unanswered questions. First, how do various network disruptions affect interest rates on loans to the nonfinancial sector, and, consequently, aggregate lending? Second, how does a particular shape of the interbank network matter for these dynamics? Third, how does the amplification and propagation of network shocks depend on the central bank policy?

I consider two scenarios for network disruption shocks: an interbank market freeze, in which all of the network connections are destroyed, and a partial network destruction, in which only a fraction of the network links ceases at the time of the shock. For both scenarios, I study two general cases for the steady-state interbank network: a complete network where all banks are connected to each other and an incomplete network in which the total number of links (connections) is below the maximum possible number of connections. I further consider three different sub-cases of an incomplete network. The first sub-case is the random network, in which banks have an equal probability of being connected to any other bank in the network. The second sub-case is the circle network, where banks have relationships with a given number of closest neighbors. Finally, I examine the scale-free network sub-case, in which only a small fraction of banks have many connections and the rest of banks have little to no connections.

In the first experiment, the economy starts with a complete interbank network and is subjected to an interbank market freeze. This shock captures a reduced level of trust in a world with asymmetric information about the counterparty risk. Following the shock, the expected cost of funding liquidity increases, resulting in an increase in the aggregate interest rate and a decrease in the aggregate lending to the real sector. However, as banks can always borrow funds from the central bank at the discount window rate, the response of the interest rate is limited by the spread between the discount window rate and the excess reserve rate (the interest rate corridor).[4] For example, if the width of the corridor is 4 percent on an annual basis, the aggregate interest rate increases by 40 basis points following the interbank market freeze. Given that the model does not feature firm or bank default, the effect on the aggregate lending is modest. In the baseline calibration, aggregate loan supply decreases by 0.25 percent on impact with the maximum decrease of 0.45 percent three periods after the shock.[5]

In the second experiment, the economy starts with a complete interbank network and is subjected to a partial network destruction. The responses of the aggregate interest rate and aggregate lending depend largely on the size of the shock. As the fraction of connections

[4]This result was first discussed in Proposition 5 in Bianchi and Bigio (2014).
[5]These values are in line with Bianchi and Bigio (2014).

destroyed at the time of the shock decreases from 100 percent (as in the interbank market freeze scenario), the response of the aggregate rate becomes less positive and switches to negative when the fraction of destroyed connections reaches a threshold value. In the baseline calibration, the threshold value is approximately 50 percent. That is, when more than a half of the links are terminated, the aggregate interest rate increases and total lending decreases, and when less than a half of the connections are destroyed, the interest rate *decreases* and lending *increases*. This result highlights that the responses of interest rates and lending are nonlinear in the size of the network disruption, implying that smaller shocks could be important to study separately from interbank market freezes.

Network shocks also have significant cross-sectional implications for the distribution of bank equity: both the total and partial destruction shocks lead to a persistent increase in the variance of equity. These distributional changes are observed because of the gradual nature of the interbank network recovery. In particular, some banks re-establish their connections sooner than others, gaining a competitive advantage in lending and accumulating equity at a faster rate. On the other hand, some banks do not regain their connections for a long time, which prevents them from lowering interest rates and increasing their supply of loans to the real sector. This, in turn, stagnates their equity growth for multiple periods. As a result, the standard deviation of the bank equity remains large for multiple periods after the shock, even when the aggregate equity reverts to its pre-shock level. This result suggests that interbank network shocks are one potential source of observed differences in bank equity.

In the third experiment, I compare the responses of variables to an interbank market freeze in the complete network to those in the incomplete network cases (random, circle, scale-free). I find that both the mean and the variance of the interest rates on loans to the nonfinancial sector are highest in the scale-free network. However, the aggregate loan rate is most responsive to the network destruction shock in the cases of random and circle networks. The distribution of bank's equity is most affected (relatively to the initial distribution) when the interbank network is scale-free.

To highlight the role of monetary policy in the transmission of network disruptions onto the real economy, I present an experiment in which I vary the width of the interest rate corridor (the difference between the discount window rate and excess reserve rate). In the complete network case, the aggregate interest rate is less sensitive to the network destruction shock when the corridor of policy rates is narrow. However, the narrower the corridor, the longer the shock affects the banks' equity. The amplification result holds for other network shapes. The differences in propagation of the shock caused by the corridor adjustment are network-specific.

The structure of the paper is as follows. Section 2 introduces the background for interbank markets and networks. Section 3 presents the model. Section 4 discusses the alternatives for the shape of the steady-state interbank network. Section 5 describes the choices for the model parameters. Section 6 presents the results. Section 7 provides policy implications. Section 8 concludes.

2 Background

In this section I discuss why interbank lending markets exist and provide a brief background about the institutional structure of these markets in order to highlight the importance of long-term bank relationships.

2.1 Interbank Lending Markets

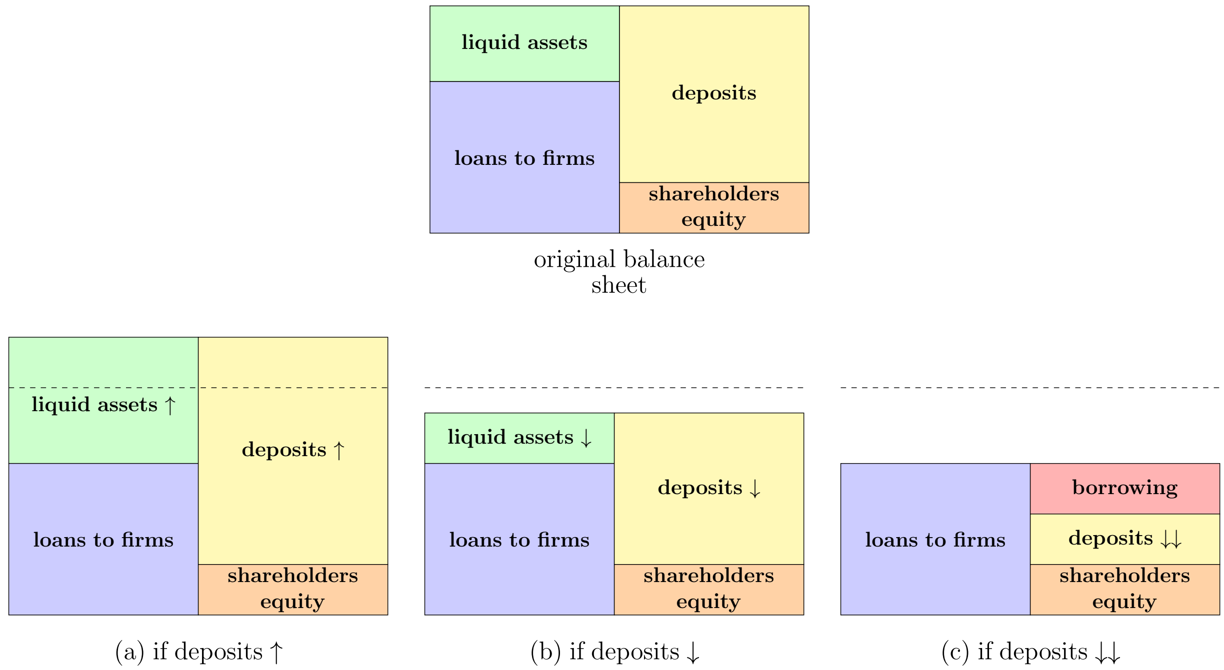
Commercial bank assets can be categorized into two general categories: liquid assets (low return) and illiquid assets (high return). Liquid assets of a typical bank include bank's reserves (vault cash and bank's deposits in its account with the central bank), short-term securities, and repurchase agreements.[6] The main categories of illiquid assets in the banking sector are commercial and industrial loans and real estate loans, more generally loans to the

[6] A repurchase agreement is a contract in which the seller of a security agrees to repurchase it from the buyer at an agreed price.

nonfinancial sector. In this paper, I categorize all assets that can be converted to cash *within a period* as liquid, and all assets that mature at some future time as illiquid. For example, if one period is a day, a loan that matures tomorrow is considered illiquid today.

The main category on the liability side of a commercial bank is the customer deposits. A large fraction of these deposits are demand deposits. The key characteristic of demand deposits is that customers can withdraw them or make additional deposits at any point during a period without a penalty. I further refer to both deposit withdrawals and additional deposits as withdrawals, with a negative withdrawal being a deposit. When a customer makes a withdrawal, the bank has to draw on its cash assets to meet this sudden demand. Every period, banks choose the portfolio shares of liquid and illiquid assets, taking into account that they may experience some withdrawals after the portfolio decision has been made.

Depending on the sign and size of deposit withdrawals, a bank can end up in three possible situations. Figure 1 illustrates these cases. If, after the portfolio decision, a bank experiences a negative withdrawal (deposit), the amount of liquid assets increases and the size of the bank's portfolio expands. This situation is referred to as having excess reserves. If a bank experiences a positive withdrawal *and* the size of the withdrawal is smaller than the amount of liquid assets, the resulting balance of liquid assets decreases and the size of bank's portfolio contracts. Finally, if the bank experiences a positive withdrawal *and* the amount of the withdrawal is greater than the amount of liquid assets on hand, the bank has a deficit of liquid assets. This situation is referred to as having a reserve deficit. To avoid a default, a bank will borrow the amount of the deficit from an available source. Banks can always take a last-resort loan from the central bank. Alternatively, they may seek a loan at a more beneficial interest rate from a bank with excess reserves. This constitutes the demand for liquidity. Why would a bank with excess reserves want to lend to a bank with a reserve deficit? Because otherwise, it earns no (or lower) interest on the excess reserves. This constitutes the supply of liquidity. Given the potential for mutually beneficial exchange, a market naturally arises.



Note: the dashed line in the three bottom panels indicates the original size of the bank's balance sheet.

Figure 1. Bank's Balance Sheet and Stochastic Withdrawals

2.2 Bank Relationships

A typical feature of an interbank lending market is that it is an over-the-counter (OTC) market, which implies that, if a bank wants to trade, it has to find a partner first. A key characteristic of the demand for liquidity is that a bank with a reserve deficit has to find the lender during the same period it experiences the deposit withdrawals that lead to the deficit. Given the OTC nature, this might be difficult to do. Additionally, banks are subject to stochastic withdrawals every period which implies that in the absence of some longer-term corresponding relationship with another bank(s), they start their search for trading partners from zero every period. By creating a network of "friends", banks can reduce this search cost. Therefore, it is beneficial for banks to build long-term relationships with each other in order to reduce the cost of finding a trading partner.

In the context of this paper, I think of a relationship between two banks as being on each

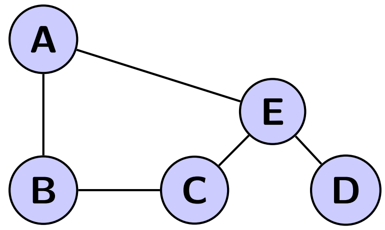


Figure 2. Example of an Undirected Graph

other's contact list. A relationship, however, is not a contract and does not obligate banks to trade with each other. It merely implies that a bank with a deficit/excess can contact another bank to check whether it has an excess or a deficit of reserves. The list of all relationships constitutes the interbank network. Note that this is not the typical definition: an interbank network is usually defined as a set of *observed* interbank trades. This distinction matters for the sequence of the events within a period.

I represent the interbank network as a graph – an ordered pair $G = (V, E)$ where V is a set of nodes (banks) and E is a set of edges (connections, links) which are two-element subsets of V. In general, graphs can be *directed* and *undirected*. An undirected graph is a graph in which edges have no orientation. For example, an undirected link between two banks implies that the two banks will trade as long as they are on the different sides of the market, whereas a directed link specifies the exact direction of the lending relationship. I will consider undirected graphs in this paper. Figure 2 shows an example of an undirected network with five vertices (banks) and five edges.

A convenient representation of a network is an *adjacency matrix* – a square $|V| \times |V|$ matrix where elements represent existing relationships between nodes. An *unweighted* adjacency matrix has elements that are either 0 or 1. The adjacency matrix of an undirected graph is symmetric. The adjacency matrix for the graph in Figure 2 is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Networks can be characterized with different measures. A *node degree*, k, is the number of nodes a node is connected to. The *degree distribution*, p_k, is the probability distribution of a node degree in a network. *Network density*, d, describes the portion of the potential connections in a network that are actual connections. Another measure of networks is the *network centralization*, which measures the extent to which the most connected node in the network is central in relation to all other nodes. The most centralized network is the star network, in which one node has the largest possible degree, $N - 1$, and all the other nodes have the smallest possible degree of 1.

Based on these measures, networks can be classified in different types. A network in which all nodes are connected to each other is a *complete network*. The density of complete networks is 100%. A network with a density lower than 100% is an *incomplete network*. Incomplete networks are further classified by their degree distribution. A network in which node degree follows the binomial distribution is a *random network*. A concept of a random network was introduced by Erdös and Rényi (1959). Random networks observe a small world property – the distance between two randomly chosen nodes in a network is short. However, random networks do not observe high clustering, which is what most real-world networks observe.[7] A network model introduced by Watts and Strogatz (1998) is an extension of the random network model that addresses the coexistence of high clustering and the small world property. It fails to explain, however, why high-degree nodes have a smaller clustering coefficient than low-degree nodes. If degree distribution follows a power law, a network is called a *scale-free network*. Scale-free networks were first discussed by Barabási and Albert (1999). Scale-free networks are commonly observed in the real world. An important property

[7] See Barabási (2016).

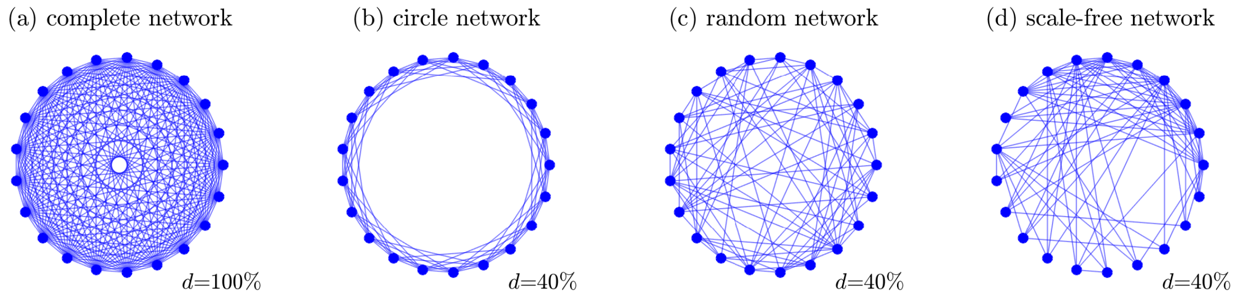


Figure 3. Network Examples

The figure displays 4 possible topologies for a network with $N = 21$ nodes. Each node represents an agent, and each line represents an existing connection between two agents. Panel (a) displays a network with the maximum number of connections, which is equal to 210. Panels (b)-(d) show different network topologies for an incomplete network that is 40% dense. The network density is displayed below each panel.

of the scale-free networks is the existence of "hubs" – nodes with an extremely high degree. Removing a hub from a network may turn the network into one with a lot of disconnected nodes. Figure 3 displays different cases for a network with 21 agents. Panel (a) depicts a complete network and Panels (b)-(c) display network topologies for a 40% dense network.

3 Model

Time is discrete, indexed by t, and has an infinite horizon. The economy consists of a representative household, an aggregate firm, a large number of banks, and a central bank. The household is risk-neutral, i.e. it is indifferent between consumption and saving. The household supplies labor to the aggregate firm. Labor is the only factor of production. The firm is subject to a working-capital constraint and needs to borrow in order to produce in every period. The firm treats loans from different banks as imperfect substitutes.

3.1 Banking Sector

The banking sector consists of a large number N of monopolistically competitive banks which are indexed by i. Banks face a perfectly elastic supply of deposits. Along with the existing

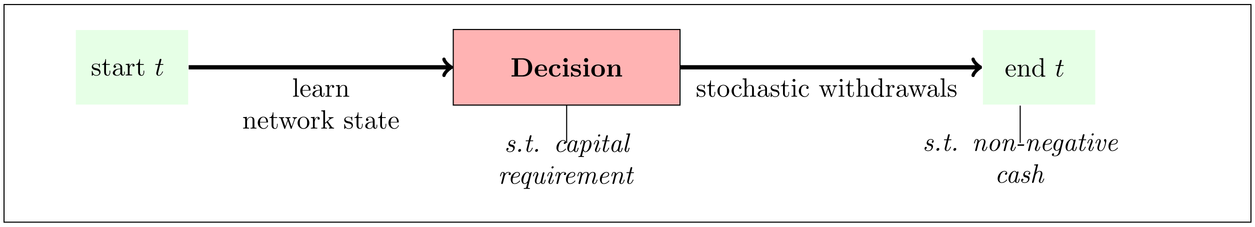


Figure 4. Sequence of Events within a Period

net worth, deposits are used to fund the bank's portfolio. There are two types of assets that are available to banks – liquid (cash assets, reserves) which earn no return and illiquid (loans to the firm) which earn interest. Banks choose the allocation of assets to maximize the expected lifetime net worth.

Banks face stochastic deposit withdrawals after they allocate their portfolio and they rely on cash assets to fulfill the demand for withdrawals. If a bank does not have enough liquid assets to meet this sudden demand, it has to borrow the amount of the deficit from either an another bank (at a low cost) or the central bank (at a high cost). The ability to borrow in the interbank market depends on whether a bank has relationships with other banks. The set of bank-to-bank relationships constitutes the *interbank network*. The interbank network is given exogenously and is subject to destruction shocks.

3.1.1 Timing and Laws of Motion

Figure 4 displays the sequence of events within a period. Starting a period with some level of net worth E_{it}, the bank i attracts household deposits D_{it} to fund new activity. The bank allocates these funds towards issuing loans B_{it} to the aggregate firm, and holding cash asset C_{it}. The balance sheet constraint is:

$$E_{it} + D_{it} = B_{it} + C_{it} \tag{1}$$

Banks earn a gross interest rate R^b_{it} on loans and pay a gross interest rate R^d (deposit rate) on the household deposits which is common for all banks and constant over time. Cash assets earn zero interest. All interest is repaid in the beginning of the next period. The amount of household deposits that a bank can hold is limited by the capital requirement:

$$D_{it} \leq \kappa_t E_{it} \tag{2}$$

where $\kappa_t \geq 1$ is a policy parameter which imposes an upper bound on the bank's debt-to-equity ratio. The capital requirement is the central bank's policy instrument that is intended to prevent banks from taking on excess leverage.

An important feature of the model is that the deposits are callable on demand, that is, at any point after the portfolio decisions have been made, the deposits may change by a random fraction ω_{it}:

$$\omega_{it} \in (-\infty, 1], \quad \omega_{it} \sim F(\cdot) \tag{3}$$

where $F(\cdot)$ is the CDF over withdrawal shocks. A negative value of ω_{it} implies that a bank realizes a random payment. I assume that deposits remain within the banking sector, that is, withdrawal shocks reshuffle deposits among banks but they do not constitute bank runs:

$$\sum_i \omega_{it} D_{it} = 0 \tag{4}$$

When a bank experiences a deposit withdrawal, it draws its cash account by the amount of the withdrawal, $\omega_{it} D_{it}$. The resulting balances of assets and liabilities are:

$$\tilde{D}_{it} = D_{it} - \omega_{it} D_{it} \tag{5}$$

$$\tilde{C}_{it} = C_{it} - \omega_{it} D_{it} \tag{6}$$

$$\tilde{B}_{it} = B_{it} \tag{7}$$

By law, banks are required to hold a minimum amount of cash assets at the end of each period. In particular, the cash-to-deposits ratio must be above a certain threshold ρ_t:

$$\tilde{C}_{it} \geq \rho_t \tilde{D}_{it} \tag{8}$$

where $\rho_t \in [0, 1]$ is the reserve requirement set by the central bank. Depending on the realization of ω_{it}, a bank can end up either with a shortage or an excess of cash assets. If a bank experiences a shortage, it must borrow the amount that is needed to meet the reserve requirement. I denote this amount by X_{it}:

$$\tilde{X}_{it} = \rho_t \tilde{D}_{it} - \tilde{C}_{it} \tag{9}$$

When $\tilde{X}_{it}$ is positive, the bank has a deficit and will take a loan from either another bank or the central bank. Banks repay these loans at the beginning of the next period. The total net cost of a reserve deficit r^x_{it} equals:

$$r^x_{it} = \begin{cases} r^{ER}_t \tilde{X}_{it} & \text{if} \quad \tilde{X}_{it} \leq 0 \text{ and excess is held at the central bank} \\ r^{DW}_t \tilde{X}_{it} & \text{if} \quad \tilde{X}_{it} > 0 \ \text{ and borrowed from the central bank} \\ r^{FF}_t \tilde{X}_{it} & \text{if} \quad \text{any } \tilde{X}_{it} \ \text{ and traded in interbank market} \end{cases} \tag{10}$$

where r^{ER}_t is the net interest rate that the central bank pays on excess reserves, r^{DW}_t is the net interest rate that the central bank charges on loans to banks (the discount window rate), and r^{FF}_t is the net interest rate that banks pay on interbank loans. The level of cash assets evolves according to:

$$C'_{it} = \tilde{C}_{it} + \tilde{X}_{it} \tag{11}$$

The bank's equity at the beginning of the next period is the sum of realized gross returns on the bank's assets and liabilities:

$$E_{it+1} = C'_{it} + R^b_{it} \tilde{B}_{it} - R^d \tilde{D}_{it} - \tilde{X}_{it} - r^x_{it} \left(\omega_{it}, D_{it}, C_{it}\right)$$

By substituting the equations (5)-(7), (9), and (11) in the above , I derive the expression for the evolution of the bank's equity as a function of the choice variables B_{it}, C_{it}, and D_{it}:

$$E_{it+1} = R^b_{it} B_{it} + C_{it} - R^d \left(1 - \omega_{it}\right) D_{it} - r^x_{it} \left(\omega_{it}, D_{it}, C_{it}\right) \tag{12}$$

The average value of r^x_{it} depends on the bank's ability to find trading partners in the interbank market, which, in turn, depends on the structure of the interbank network.

3.1.2 Interbank Market

Banks can enter the interbank market for two reasons: banks with a cash shortage seek to find a loan at a rate lower than r^{DW}_t and banks with an excess of cash seek to earn a rate higher than r^{ER}_t. I assume that an interbank loan cannot exceed the amount of the bank's reserve deficit. Thus, the only purpose of the interbank market in this model is for redistribution of reserves. A bank with an excess places lending orders and a bank with a deficit places borrowing orders. Following Atkeson et al. (2012) and Bianchi and Bigio (2014), I assume that a bank places a continuum of orders of infinitesimal size (further referred to as $1), and the trades occur on a dollar-per-dollar basis. The interest rate at which a trade occurs, r^{FF}_t, is determined by the bilateral Nash bargaining.

Problem 1 *The bargaining problem between a lending order and a borrowing order is:*

$$\max_{r^{FF}_t} \left(r^{DW}_t - r^{FF}_t\right)^{\xi} \left(r^{FF}_t - r^{ER}_t\right)^{1-\xi}$$

where ξ is the bargaining power of the borrowing order.

The first-order condition implies that the interbank loan rate is a convex combination of the two policy rates:

$$r^{FF}_t = \xi r^{ER}_t + (1 - \xi) r^{DW}_t \tag{13}$$

Note that for any Nash bargaining parameter, $r^{ER}_t \leq r^{FF}_t \leq r^{DW}_t$; therefore, banks with a cash deficit will always try to borrow in the interbank market before borrowing from the central bank. Similarly, banks with excess reserves will always try to lend in the interbank market. I assume that all borrowing orders have the same bargaining power, which implies that the interbank lending rate is identical for all matched orders. The probabilities of finding a matching order, however, can vary across banks, depending on the bank's position in the interbank network.

3.1.3 Interbank Network

An important feature of the interbank market is that, in order to trade, banks have to search for a trading partner. This gives rise to an interbank network. I define the interbank network as in Lenzu and Tedeschi (2012). Banks enter bilateral potential trading agreements (PTAs). These agreements constitute a promise to engage in trade when one of the banks has a cash surplus and another has a cash deficit. If the two banks hold a PTA bt end up on the same side of the market (both are lenders or both are borrowers), the PTA can not be enforced.

Definition 1 *The interbank network is an undirected graph* (N, G) *where* $N = [1, ..., n]$ *is the set of nodes (banks) and* G *is the* $n \times n$ *symmetric adjacency matrix with elements* $G_{ijt} \in \{0, 1\}$ *that represent a relationship between banks* i *and* j *in the following way:*

$$G_{ijt} = G_{jit} = \begin{cases} 1 & \text{if there exists a PTA between } i \text{ and } j \\ 0 & \text{if there is no PTA between } i \text{ and } j \end{cases}$$

where $G_{ii} = 0 \; \forall \; i$*, i.e. a bank cannot be connected to itself.*

The adjacency matrix G_t evolves exogenously and is known in the beginning of a period. If every bank has a PTA with every other bank in the network, then the interbank network is complete; if there are no PTAs between any banks, then the interbank network is empty. When the network is incomplete, banks have different probabilities of finding a trading partner in the interbank market.

Consider a bank i with a deficit of reserves. What is the probability that i can get a loan from another bank? It depends on (1) how many banks with a cash surplus i is connected to (lending neighbors) and (2) how many banks with a cash deficit i's lending neighbors are connected to (borrowing neighbors). Recall that a bank i has a cash deficit if $X_{it} > 0$ and a cash surplus otherwise. The mass of lending that is available to i is the sum of cash surpluses from the i's lending neighbors:

$$\Upsilon_i^+ = \sum_j G_{ijt} X_{jt} I(X_{jt} \leq 0) \tag{14}$$

where i's neighbors are indexed by j and $I(\cdot)$ is the indicator function. The mass of borrowing that is available to the i's lending neighbors is:

$$\Upsilon_i^- = \sum_k I(K_{kt} \geq 1)\, X_{kt} I(X_{kt} > 0)\,, \quad \underset{1\times N}{K} = \sum_j G_{ijt} G_{jt} \tag{15}$$

where the neighbors of i's neighbors are indexed by k. I assume that borrowing and lending orders are paired at random. If $\Upsilon_i^+ < \Upsilon_i^-$, then there is more borrowing orders than lending orders in the i's neighborhood implying that some of the borrowing orders will not be matched. If $\Upsilon_i^+ \geq \Upsilon_i^-$, however, all borrowing orders will be matched.[8] The probability that the bank i can fund its cash deficit in the interbank market equals to:

$$p_{it}^{BL} = \min\left\{1, \frac{\mathbb{E}\left[\Upsilon_i^+\right]}{\mathbb{E}\left[\Upsilon_i^-\right]}\right\} = \min\left\{1, \frac{\sum_j G_{ijt} P(X_{jt} \leq 0)\, \mathbb{E}[X_{jt}|X_{jt} \leq 0]}{\sum_k I(K_{kt} \geq 1)\, P(X_{kt} > 0)\, \mathbb{E}[X_{kt}|X_{kt} > 0]}\right\} \tag{16}$$

and the probability that i will borrow from the central bank is $1 - p_{it}^{BL}$. The expected per-unit cost of a reserve deficit for a bank i:

$$\chi_{it}^B = p_{it}^{BL} r_t^{FF} + \left(1 - p_{it}^{BL}\right) r_t^{DW} \tag{17}$$

[8]This type of matching was used Bech and Monnet (2014) and Bianchi and Bigio (2014)

Similarly, if i is a lending bank, the probability of its lending that is matched in the interbank market is:

$$p_{it}^{LB} = \min\left\{1, \frac{\mathbb{E}\left[\Gamma_i^-\right]}{\mathbb{E}\left[\Gamma_i^+\right]}\right\} = \min\left\{1, \frac{\sum_j G_{ijt}\; P(X_{jt} > 0)\, \mathbb{E}[X_{jt}|X_{jt} > 0]}{\sum_k I(K_{kt} \geq 1)\, P(X_{kt} \leq 0)\, \mathbb{E}[X_{kt}|X_{kt} \leq 0]}\right\} \tag{18}$$

where Γ_i^- is the mass of borrowing that is available from the i's borrowing neighbors and Γ_i^+ is the mass of lending available to the i's borrowing neighbors. The expected return on a unit of the i's cash surplus equals to:

$$\chi_{it}^{L} = p_{it}^{LB} r_t^{FF} + \left(1 - p_{it}^{LB}\right) r_t^{ER} \tag{19}$$

In general, $\Upsilon_i^+ \neq \Gamma_i^+$ and $\Upsilon_i^- \neq \Gamma_i^-$, meaning that the deposit withdrawals cannot be perfectly insured against. This is because banks can trade only with banks that they are connected to. As a result, the amunt of the total interbank trading is weakly less than the aggregate cash deficit in the banking sector. Intuitively, in such setup it is more burdensome for a bank to end up with a reserve deficit compared to the case where all banks are connected to each other. Equation (10) which defines the total cost of a reserve deficit can be rewritten in terms of χ_{it}^{L} and χ_{it}^{B}:

$$r_{it}^{x} = \begin{cases} \chi_{it}^{L}\left[(\rho_t + (1-\rho_t)\omega_{it})D_{it} - C_{it}\right] & \text{if} \quad (\rho_t + (1-\rho_t)\omega_{it})D_{it} - C_{it} \leq 0 \\ \chi_{it}^{B}\left[(\rho_t + (1-\rho_t)\omega_{it})D_{it} - C_{it}\right] & \text{if} \quad (\rho_t + (1-\rho_t)\omega_{it})D_{it} - C_{it} > 0 \end{cases} \tag{20}$$

This cost has a discontinuity at the point where the bank's deficit is zero. This occurs when the value of the withdrawal shock equals to:

$$\omega_{it}^{*} = \left(\frac{C_{it}}{D_{it}} - \rho_t\right) / (1 - \rho_t) \tag{21}$$

If the realized value of the withdrawal shock is below the threshold value ω_{it}^{*}, then the bank has excess reserves, which can be lent out at χ_{it}^{L}. If the realized value of the withdrawal shock is above ω_{it}^*, then the bank has a reserve deficit and has to get a loan at the cost χ_{it}^B. By rewriting the expression for the bank's deficit in terms of ω_{it}^* and taking an expectation over the withdrawal shocks, I find the expected total net cost of a cash deficit is:

$$\mathbb{E}_\omega r_{it}^x = (1-\rho_t) D_{it} \left[\chi_{it}^B \left(\bar{\omega} - \omega_{it}^* \right) + \left(\chi_{it}^B - \chi_{it}^L \right) \left(\omega_{it}^* F\left(\omega_{it}^* \right) - \int_{-\infty}^{\omega_{it}^*} \omega f(\omega) d\omega \right) \right] \tag{22}$$

where $\bar{\omega}$ is the mean of withdrawal shocks and $f(\omega)$ is the PDF over withdrawal shocks. If the chosen ratio of cash to deposits C_{it}/D_{it} is below the reserve requirement ρ_t, then the future value of the withdrawal shock must be negative, i.e. bank must experience a deposit, for a bank not to have a reserve deficit. If, however, the bank chooses to hold higher than required cash-to-deposits ratio, it may still end up with a non-negative cash balance, even if it experiences a withdrawal.

3.1.4 Bank's Problem

Banks maximize the expected net worth (equity) subject to the balance sheet constraint and the capital requirement:

Problem 2 *Banks solve the following maximization problem:*

$$\begin{aligned}
& \max_{D_{it}, B_{it}, C_{it}} \mathbb{E}_t \sum_{j=1}^{\infty} (\beta\gamma)^j \Lambda_{t+j} E_{it+j} \\
s.t. \quad & E_{it+1} = R_{it}^b B_{it} + C_{it} - R^d \left(1 - \omega_{it}\right) D_{it} - r_{it}^x \left(\omega_{it}, D_{it}, C_{it}\right) \\
& E_{it} = B_{it} + C_{it} - D_{it} \\
& D_{it} \leq \kappa_t E_{it} \\
& B_{it}, C_{it}, D_{it} \geq 0
\end{aligned}$$

where $\beta \Lambda_t$ is the household's stochastic discount factor and γ is the additional impatience parameter for banks. I substitute the expression for the evolution of equity into the objective and write the bank's value function at time t as follows:

$$
\begin{aligned}
V_t = \max_{D_{it},B_{it},C_{it}} \; & \mathbb{E}\,\beta\gamma\Lambda_t \left[R^b_{it}B_{it} + C_{it} - R^d\left(1-\omega_{it}\right)D_{it} - r^x_{it}\left(\omega_{it},D_{it},C_{it}\right)\right] \\
\text{s.t.} \qquad & E_{it} = B_{it} + C_{it} - D_{it} \\
& D_{it} \leq \kappa_t E_{it} \\
& B_{it}, C_{it}, D_{it} \geq 0
\end{aligned}
$$

Further substitution and simplification results in:

$$
\begin{aligned}
V_t = \max_{D_{it},C_{it}} \; & \beta\gamma\Lambda_t \left[R^b_{it}\left(E_{it} - C_{it} + D_{it}\right) + C_{it} - R^d\left(1-\bar{\omega}\right)D_{it} - \mathbb{E}_\omega r^x_{it}\left(D_{it},C_{it}\right)\right] \\
\text{s.t.} \qquad & D_{it} \leq \kappa_t E_{it} \\
& B_{it}, C_{it}, D_{it} \geq 0
\end{aligned}
$$

Since E_{it} is known at the time of decision, solving for the loans, reserves, and deposits as fractions of equity is equivalent to solving the original bank's problem. I define the ratios:

$$
\left[d_{it} \;\; b_{it} \;\; c_{it} \;\; e_{it+1}\right] \equiv \left[\frac{D_{it}}{E_{it}} \;\; \frac{B_{it}}{E_{it}} \;\; \frac{C_{it}}{E_{it}} \;\; \frac{E_{it+1}}{E_{it}}\right] \tag{23}
$$

where e_{it+1} is the growth rate of bank's equity. Rewriting the bank's problem in terms of equity shares:

$$
\begin{aligned}
\max_{d_{it},c_{it}} \quad & R^b_{it} - \left(R^b_{it} - 1\right)c_{it} + \left(R^b_{it} - \left(1-\bar{\omega}\right)R^d\right)d_{it} - \mathbb{E}_\omega r^x_{it}\left(d_{it},c_{it}\right) \\
\text{s.t.} \qquad & d_{it} \leq \kappa_t \\
& c_{it}, d_{it} \geq 0
\end{aligned}
$$

Next, I define the cash-to-deposits ratio (liquidity ratio), L_{it}:

$$L_{it} = \frac{c_{it}}{d_{it}} \tag{24}$$

and transform the bank's problem as follows:[9]

$$R^b_{it} + \max_{d_{it}\in[0,\kappa_t]} d_{it}\left[R^b_{it} - (1-\bar{\omega})\,R^d + \max_{L_{it}\in[0,1+d_{it}]}\left\{-\left(R^b_{it}-1\right)L_{it} - \mathbb{E}_\omega r^x_{it}\,(1,L_{it})\right\}\right]$$

The above shows that the bank solves two separate problems: first, it chooses the fraction of deposits that it will hold in cash assets, and then it picks the optimal leverage scale. The non-standard feature of the liquidity-management problem is that the cost of the cash deficit, r^x_{it}, has a discontinuity at the point ω^*_{it}. Differentiating the objective with respect to L_{it} gives the bank i's loan supply equation:

$$R^b_{it} - 1 = -\frac{\partial \mathbb{E}_\omega r^x_{it}\,(1,L_{it})}{\partial L_{it}} \tag{25}$$

where

$$\mathbb{E}_\omega r^x_{it}\,(1,L_{it}) = (1-\rho_t)\left[\chi^B_{it}\,(\bar{\omega}-\omega^*_{it}) + \left(\chi^B_{it}-\chi^L_{it}\right)\left(\omega^*_{it}F\,(\omega^*_{it}) - \int_{-\infty}^{\omega^*_{it}} \omega f(\omega)d\omega\right)\right]$$

and $\omega^*_{it} = (L_{it}-\rho_t)\,/(1-\rho_t)$. The left-hand side of equation (25) is the opportunity cost of holding cash assets over loans and the right-hand side is the expected benefit of holding an additional unit of cash asset. This optimality condition provides a unique solution for L_{it}. Given the optimal level of L_{it}, the return on additional unit of leverage is:

$$R^L_{it} = R^b_{it} - (1-\bar{\omega})\,R^d - \left(R^b_{it}-1\right)L^*_{it} - \mathbb{E}_\omega r^x_{it}\,(1,L_{it}) \tag{26}$$

The leverage scale problem is linear in R^L_{it}. Thus, if this return is positive, the bank will pick

[9] Since $\omega_{it} \leq 1$, then it must be that $L_{it} = \frac{c_{it}}{d_{it}} \leq 1$. $b_{it} = 0$ is ruled out by the shape of the loan demand.

the maximum possible share of deposits, κ_t. Otherwise, the bank will choose to hold zero household deposits. The expected return on the optimal portfolio is then:

$$\mathbb{E}_\omega e^*_{it+1} = R^b_{it} + \max\left\{0, \kappa_t R^L_{it}\right\} \tag{27}$$

Once the optimal portfolio shares are found, the optimal levels of loans, cash, and deposits can be calculated as follows:

$$[D^*_{it} \ \ B^*_{it} \ \ C^*_{it}] = E_{it} \times [d^*_{it} \ \ b^*_{it} \ \ c^*_{it}] \tag{28}$$

This concludes the bank's problem.

3.2 Real Sector

The representative household obtains utility from consumption, C_t, and disutility from labor, H_t. The household can save by supplying deposits D^A_t to the banking sector.

Problem 3 *The household solves the following maximization problem:*

$$\begin{aligned} \max_{C_t, H_t, D^A_t} \quad & \sum_{t=0}^{\infty} \beta^t \left[C_t - \frac{H_t^{1+\nu}}{1+\nu}\right] \\ s.t. \quad & D^A_t + C_t = W_t H_t + R^d D^A_{t-1} + \Pi_t + T_t \end{aligned}$$

where W_t is the real wage rate, Π_t is the firm's profit, T_t is the tax transfer, and ν is the inverse of the Frisch elasticity. The labor supply curve is:

$$H_t = W_t^{\frac{1}{\nu}} \tag{29}$$

which implies that the household's total wage income is $W_t^{\frac{\nu+1}{\nu}}$. If $R^d = \frac{1}{\beta}$, the household is indifferent between consumption and saving, and:

$$C_t \in [0, Y_t], \quad D_t^A = Y_t - C_t \tag{30}$$

where Y_t is the output of the aggregate firm.

The profit-maximizing firm uses the household's labor to produce output according to the following production function:

$$Y_t = A_t H_t^{1-\alpha} \tag{31}$$

where A_t is a technology index, and $1-\alpha$ is the labor share. The firm has to pay workers before the output is realized, therefore, to cover the wage bill, it borrows the amount I_t^A from the banking sector:

$$W_t H_t = I_t^A \tag{32}$$

I_t^A is collected via the CES technology:

$$I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{33}$$

where I_{it} is borrowing from bank i, λ_i is the bank i's share, and ϵ is the elasticity of substitution between loans from different banks. The firm repays the loan principal and accrued interest in the beginning of the next period. The total repayment to the banking sector is $\sum_i R_{it}^b I_{it}$. The firm never defaults on its loans.

Problem 4 *The aggregate firm solves the following maximization problem:*

$$\begin{aligned}
\max_{I_t^A, I_{it}, H_t} \quad & \sum_{t=0}^{\infty} \beta^t \left[A H_t^{1-\alpha} - W_t H_t + I_t^A - \sum_i R_{it-1}^b I_{it-1} \right] \\
s.t. \quad & W_t H_t = I_t^A \\
& I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}$$

By taking the first-order conditions and imposing the labor market clearing, I find the demand curve for a loan from the bank i:

$$R_{it}^b = \frac{(1-\alpha) A_t}{\beta} \left[I_t^A \right]^{\frac{1}{\epsilon} - \frac{\nu+\alpha}{\nu+1}} \left[\frac{I_{it}}{\lambda_i} \right]^{-\frac{1}{\epsilon}}, \tag{34}$$

the aggregate repayment of loans to the banking sector:

$$\sum_i R_{it}^b I_{it} = \frac{(1-\alpha) A_t}{\beta} \left[I_t^A \right]^{\frac{1-\alpha}{\nu+1}} = \frac{1-\alpha}{\beta} Y_t \tag{35}$$

and the firm's profit is:

$$\Pi_t = A_t H_t^{1-\alpha} - \sum_i R_{it-1}^b I_{it-1} = Y_t - \frac{1-\alpha}{\beta} Y_{t-1} \tag{36}$$

Refer to Appendix D for the details of derivations.

3.3 Central Bank

The central bank starts a period with M_t^0 reserves and D_t^{CB} bank deposits. It issues new reserves, Δ_t^{CB}, receives interest on discount window loans, pays interest on excess reserves, and makes a transfer T_t to the household. The laws of motion for the deposits and reserves respectively are:

$$D_{t+1}^{CB} = D_t^{CB} + \Delta_t^{CB} + r_t^{DW} X_t^- - r_t^{ER} X_t^+ - T_t \tag{37}$$

$$M_{t+1}^0 = M_t^0 + \Delta_t^{CB} \tag{38}$$

where X_t^- is the total amount of loans to the banking sector and X_t^+ is the aggregate excess reserves held at the central bank. By combing the laws of motion, I derive the central bank's budget constraint:

$$M_{t+1}^0 - M_t^0 = D_{t+1}^{CB} - D_t^{CB} - r_t^{DW} X_t^- + r_t^{ER} X_t^+ + T_t \tag{39}$$

The central bank's policy rates satisfy:

$$r_t^{DW} \geq r_t^{ER}$$

The difference between the two rates constitutes the interest rate corridor. The central bank chooses r_t^{DW} and r_t^{ER} to target a particular level of the interbank loan interest rate, r_t^{FF}.

3.4 Exogenous Processes

The interbank network is given exogenously. I assume that there exists a steady-state interbank network and that the degree distribution of bank's connections is orthogonal to the distribution of the withdrawal shocks.

3.4.1 Steady-State Interbank Network

I consider two general cases for the steady-state interbank network: a complete interbank network and an incomplete interbank network. For the incomplete case, I consider three subcases: a random interbank network, a circle interbank network, and a scale-free interbank network. For each case, I find how centralized the interbank network is. I measure network centralization with the Freeman's centralization index:

$$C_k = \frac{\sum_i \left(k^{\max} - k_i\right)}{\left(N-1\right)\left(N-2\right)} \tag{40}$$

where the numerator is the sum of differences between the highest degree in the network and degrees of other banks and the denominator is the maximum centralization that can be attained in a graph with N nodes. I discuss each network case below and summarize the cases in Table 1.

Table 1. INTERBANK NETWORK TOPOLOGY CASES

Network	Degree distribution	K	$\bar{k}$	C_k
Complete	Dirac delta centered at $N-1$	$\frac{N(N-1)}{2}$	$N-1$	0
Circle	Dirac delta centered at k^*	$Nk^*/2$	k^*	0
Random	binomial with $p = k^*/(N-1)$	$Nk^*/2$	k^*	low
Scale-free	power law with $\gamma = 3,\ m = k^*/2$	$Nk^*/2$	k^*	high

In the complete network, all banks are connected to each other. The total number of connections K and the bank degree k take their maximum possible values,

$$K^C = K^{\max} = N\left(N-1\right)/2 \tag{41}$$

$$k_i^C = k^{\max} = N-1 \tag{42}$$

respectively. Here, all banks have an equal number of connections and the network centralization index is zero.

In the incomplete network, not all banks are connected to each other, i.e. the number of connections is below the network's capacity:

$$K^I = \frac{\bar{k}N}{2} < K^{\max} \tag{43}$$

where $\bar{k}$ is the average degree of a bank. Here, individual banks may differ in their degree. I consider three different cases of the incomplete network. For comparison, I set the average degree $\bar{k}$ to equal k^* for all the sub-cases, such that the total number of connections is the same in each network.

In the circle network, each bank is connected to k^* closest banks and the total number of links is $Nk^*/2$. Degree distribution of a circle network is a Dirac delta function centered at k^*. Since all banks have the same number of connections, the centralization index of the circle network is zero.

The degree distribution of a random network is binomial and the probability that a bank is connected to k other banks is:

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k} \tag{44}$$

The average degree of a random network is $\bar{k} = p(N-1)$ and the centralization index of a random network tends to be close to zero.

The degree distribution of the scale-free network follows a power law and the probability that a bank is connected to k other banks is:

$$p_k = ak^{-\gamma}, \quad 2 < \gamma \leq 3 \tag{45}$$

where a is a normalization constant.[10] By the result in Klemm and Eguiluz (2002), the average degree of a scale-free network is:

$$\bar{k} = \sqrt{2a} \tag{46}$$

Compare to other types of the incomplete network, scale-free networks observe high network centralization.

I denote the degree distribution of the steady-state network by p_k^{ss}. The steady-state

[10]Follows from a Barabási-Albert model where $a = 2m^2$. Given an initial network with m_0 nodes, m new nodes is added to each node at each step until the network reaches a size N. Typically, $m_0 = m$. See Barabási and Albert (1999) for more details.

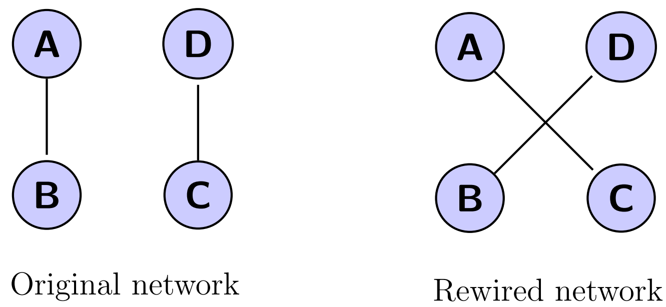


Figure 5. Single Iteration of a Degree-Preserving Randomization Algorithm for a Four-Bank Network

network at time t is a particular realization of the network implied by p_k^{ss}. The number of connections in this network is:

$$K^{ss} = \frac{1}{2}\sum_{i=1}^{N} k_i^{ss}, \quad K^{ss} \leq K^{\max} = N\left(N-1\right)/2 \tag{47}$$

where k_i^{ss} is the steady-state degree of the bank i.

I use the algorithm proposed in Maslov and Sneppen (2002) to generate the steady-state network. Each network realization is random, however, all nodes have the same degrees as in the original (null) network. The algorithm works as follows. First, a random edge (i, j) is selected from the null network. Next, a second random edge (u, v) is selected such that $i \neq u$, $j \neq v$, and edges (i, v) and (j, u) do not already exist in the network. Then, edges (i, j) and (u, v) are removed from the network and edges (i, v) and (j, u) are added. This process is repeated until each link in the null network is rewired at least once. Figure 5 demonstrates an example for a single iteration of this algorithm.

3.4.2 Interbank Network Shocks

At the time of a shock, a fraction $\zeta \in [0, 1]$ of K^{ss} links is removed. In the baseline model, the subset of links that are removed is chosen at random. The number of remaining connections at the time of the shock is $(1-\zeta)\,K^{ss}$. In the next period, a fraction of the destroyed connections $s \in [0,1]$ are reestablished. Here, $s \in [0,1]$ corresponds to the speed with which the network is rebuilt. The resulting number of links one period after the shock is:

$$K_{t+1} = (1-\zeta)\,K^{ss} + s\zeta K^{ss} \tag{48}$$

A list with $s\zeta K^{ss}$ of new edges is generated according to the degree-preserving randomization process. At $t+2$, if the number of existing connection is below the original number of links, $s(1-s)\zeta K^{ss}$ of links are rebuilt. Otherwise, , no new connections are created. A general formula for the total number of links in the network j periods after the shock is:

$$K_{t+j} = \begin{cases} K_{t+j-1} + s\sum_{n=0}^{j}(-s)^n\zeta K^{ss} & \text{if} \quad L_{t+j-1} < K^{ss} \\ K^{ss} & \text{if} \quad L_{t+j-1} = K^{ss} \end{cases} \tag{49}$$

3.4.3 Withdrawal Shocks

I assume that withdrawal shocks ω_{it} are drawn from the logistic distribution $F^L(\omega)$ with the mean $\bar{\omega}$ and the standard deviation σ:

$$F^L(\omega) = \frac{1}{1+e^{\frac{\bar{\omega}-\omega}{\sigma}}}$$

To account for the fact that ω_{it} is a fraction that takes a maximum value of 1, I truncate the distribution $F^L(\omega)$ at 1:

$$F(\omega) = \frac{F^L(\omega)}{F^L(1)}$$

I assume that $F(\omega)$ is time-invariant.

3.5 Market Clearing

Deposit Market. Supply of deposits must equal demand for deposits:

$$D_t^A = \sum_i D_{it} \tag{50}$$

Money market. Cash assets held by banks during the decision stage must equal the central bank's supply of money:

$$\sum_i C_{it} = M_t^0 \tag{51}$$

The total amounts borrowed and held at the central bank respectively are:

$$X_t^- = \sum_i I\left(\tilde{X}_{it} > 0\right) \tilde{X}_{it} \left(1 - p_{it}^{BL}\right) \tag{52}$$

$$X_t^+ = \sum_i \left(1 - I\left(\tilde{X}_{it} > 0\right)\right) \tilde{X}_{it} \left(1 - p_{it}^{LB}\right) \tag{53}$$

Loan Market. The supply of loans from the bank i equals to the firm's demand for loans from the bank i:

$$B_{it}^S = B_{it}^D \tag{54}$$

Appendix E shows the full list of equilibrium conditions.

4 Theoretical Analysis

I define the net loan rate that bank i charges the firm as $r_{it}^b = R_{it}^b - 1$. The optimality condition (25) can be rewritten as:

$$r_{it}^b = \chi_{it}^B - \left(\chi_{it}^B - \chi_{it}^L\right) F(\omega_{it}^*) \tag{55}$$

where $\omega_{it}^{*} = (L_{it} - \rho_t) / (1 - \rho_t)$ is the threshold value of the withdrawal shock that makes the bank's deficit equal zero. Rewriting χ_{it}^{B} and χ_{it}^{L} in terms of the policy rates r_t^{ER} and r_t^{DW} results in:

$$\chi_{it}^{B} = \xi p_{it}^{BL} r_t^{ER} + \left(1 - \xi p_{it}^{BL}\right) r_t^{DW} \tag{56}$$

$$\chi_{it}^{L} = \left(1 - (1 - \xi) p_{it}^{LB}\right) r_t^{ER} + (1 - \xi) p_{it}^{LB} r_t^{DW} \tag{57}$$

I substitute the above definitions into the optimality condition (55) to express the loan rate as a function of policy rates:

$$r_{it}^{b} = r_t^{DW} - \left(r_t^{DW} - r_t^{ER}\right) \left[\underbrace{F\left(\frac{L_{it}-\rho_t}{1-\rho_t}\right) \left(1-(1-\xi) p_{it}^{LB}\right)}_{\substack{\text{prob. of interbank match} \\ \text{if lender}}} + \underbrace{\left(1-F\left(\frac{L_{it}-\rho_t}{1-\rho_t}\right)\right) \xi p_{it}^{BL}}_{\substack{\text{prob. of interbank match} \\ \text{if borrower}}} \right] \tag{58}$$

Holding the cash-to-deposits ratio constant, there are three exogenous factors that affect the loan rate. First, the interest rate on the loans to the real sector is bounded above by the discount window rate r_t^{DW}. The lower the discount window rate is, the less costly the loans are to the firm. Second, the corridor between the discount window rate and the excess reserves rate, $r_t^{DW} - r_t^{ER}$, determines how low the loan rate can be. If the width of the corridor is zero, than the loan rate is set at the discount window rate level. As the difference between the two policy rates increases, all else equal, the loan rate decreases. Finally, the loan rate depends on the probabilities of finding a trading partner in the interbank market, p_{it}^{LB} and p_{it}^{BL}. In general, these probabilities do not move proportionally to each other – sometimes both will increase or decrease and sometimes they will move in the opposite directions, depending on the state of the interbank network. It is useful to consider the limiting cases for the interbank network.

When the network is empty, banks have a zero chance of finding a trading partner in the interbank network, that is $p_{it}^{LB} = p_{it}^{BL} = 0$. The loan rate in this case is common across banks and equals to:

$$r_t^{b,E} = r_t^{DW} - \left(r_t^{DW} - r_t^{ER}\right) F\left(\tfrac{L_{it}-\rho_t}{1-\rho_t}\right) \tag{59}$$

When the network is complete, the matching probabilities are common across banks but they are not necessarily equal to each other:

$$p_{it}^{LB} = p_t^{LB} = \min\{1, \Psi\} \quad \text{and} \quad p_{it}^{BL} = p_t^{BL} = \min\left\{1, \frac{1}{\Psi}\right\}$$

where

$$\Psi \equiv -\frac{(1-F(\omega_t^*)) \int_{\omega_t^*}^{1} (\omega - \omega_t^*) f(\omega) d\omega}{F(\omega_t^*) \int_{-\infty}^{\omega_t^*} (\omega - \omega_t^*) f(\omega) d\omega} \geq 0$$

is the total mass of borrowing orders relative to the total mass of lending orders in the banking sector. If $\Psi > 1$, then, on aggregate, there is an expected cash deficit in the banking sector. This happens when, on average, banks choose to set their cash-to-deposits ratio at the level that is smaller than the reserve requirement ρ_t. In this case, the loan rate in the complete network will be higher than it is in the empty network. If, $\Psi \leq 1$, however, the loan rate in the complete network will always be lower than it is in the empty network. Figure 6 displays these cases for a given choice of L_t. As Ψ increases, the loan rate in the complete network decreases.

When the interbank network is incomplete, the probabilities p_{it}^{LB} and p_{it}^{BL} will vary with the type of the network. Their changes may have an asymmetric effect on the loan rate. Depending on the different combinations of the two probabilities, the loan rate can be above, below, or equal to the loan rate in the empty or complete interbank networks. Figure 7 shows the possible values of the loan rate for different pairs of p_{it}^{LB} and p_{it}^{BL}.

5 Quantitative Exercises

I set the household's discount factor to 1, which implies that the gross interest rate on the household deposits is 1. I assume a constant CES share for each bank, $\lambda = 1/N$. I set

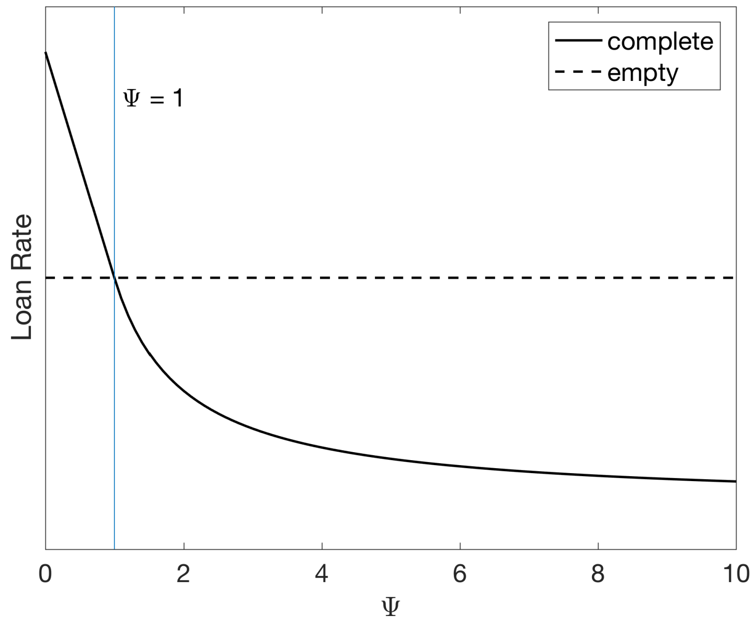


Figure 6. Loan Rate as a Function of Ψ

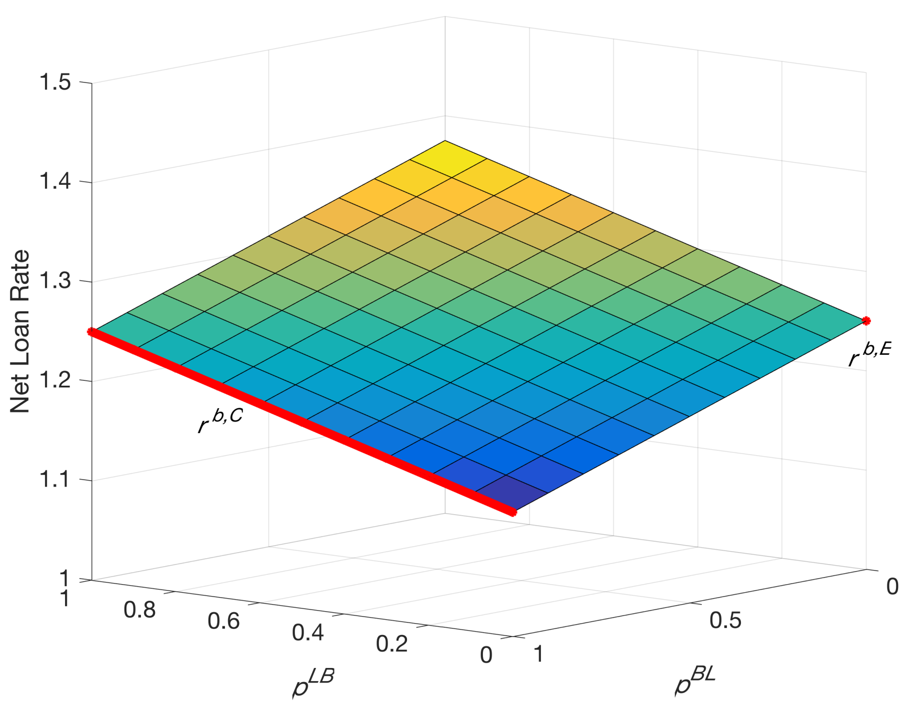


Figure 7. Loan Rate as a Function of Interbank Matching Probabilities

Table 2. PARAMETER VALUES

Parameter	Value	Definition
β	1.00	household's discount factor
α	0.00	capital input share
$1/\nu$	2.50	Frisch elasticity of labor supply
A_{ss}	1.00	steady-state firm productivity
ϵ	5.00	elasticity of substitution between loans
λ	$1/N$	bank's share in loan aggregator
ξ	0.50	bargaining power of a borrower in the interbank market
$\bar{\omega}$	0.00	mean of withdrawal shocks
σ	0.10	standard deviation of withdrawal shocks
r_t^{ER}	0.00%	annual interest rate on excess reserves
r_t^{DW}	2.50%	annual discount window rate
r_t^{FF}	1.25%	annual target interbank loan rate
N	100.00	number of banks
ζ	0.50	fraction of links destroyed at the time of the shock
s	0.25	fraction of destroyed connections rebuilt each period after the shock
k^*	0.20	average degree of a bank in incomplete network
κ_{ss}	10.00	capital requirement to match 10% capital ratio
L_{ss}	5.00%	steady-state cash-to-deposits ratio
γ	0.97	bank's discount factor
ρ_{ss}	0.05	reserve requirement

the annual net interest rate paid on excess reserves to $r_t^{ER} = 0$ and the annual discount window rate to $r_t^{DW} = 2.5\%$. I assume equal bargaining power of borrowers and lenders in the interbank market, which implies the value of $\xi = 0.5$ and the central bank's target for the interbank loan rate $r_t^{FF} = 1.25\%$. I set $\kappa_{ss} = 10$, such that the steady-state deposit-to-equity ratio is 10%. I set the reserve requirement to 5% and the bank's discount factor at $\gamma = 0.97$. Table 2 states the full list of parameters.

5.1 Static Network

Figure 8 displays the distributional properties of the loan rates and equity growth rates for the complete, circle, random, and scale-free interbank networks. The incomplete networks exhibit more variation in both the loan rates and the equity growth rates, compared to those in the complete network case. As the incomplete network becomes more centralized, the dispersion of both loan rates and equity growth rates increases, reaching its largest value

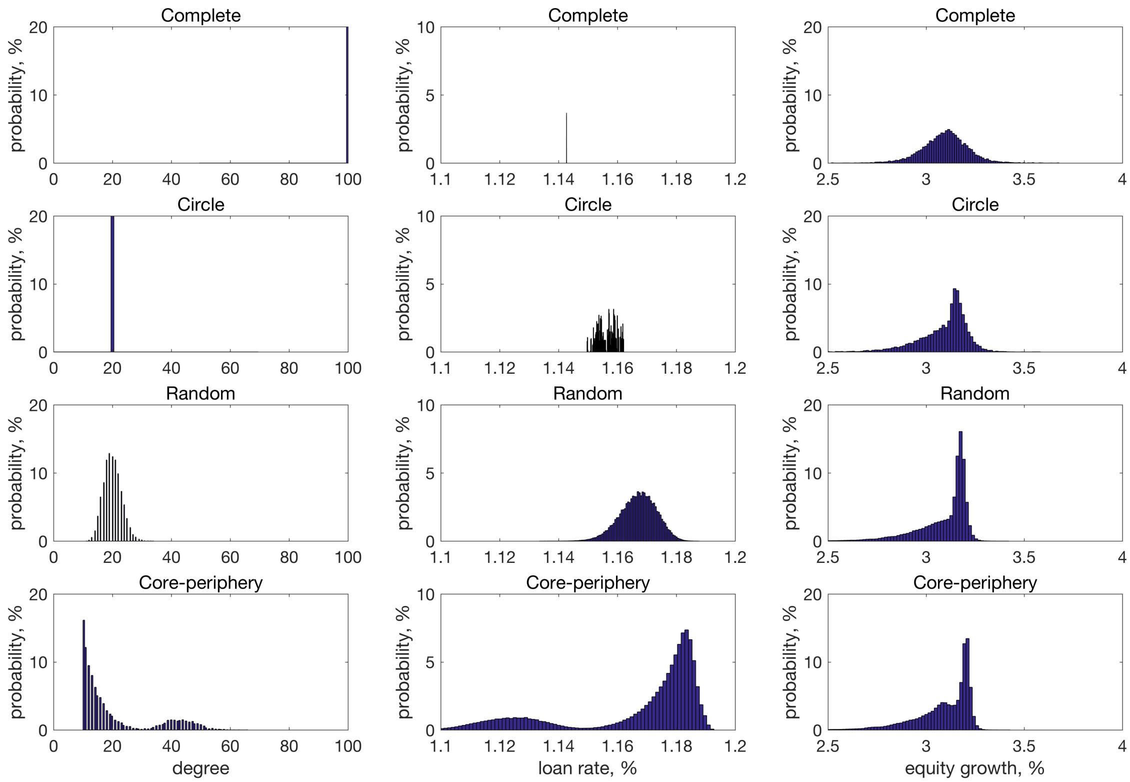


Figure 8. Empirical Distribution of Interest Rates and Equity Growth Rates

The figure displays results for 4 different cases of interbank network with 100banks based on 10,000 simulations. The left panel shows the distribution of bank's network degree, the middle panel displays the net annualized loan rates, expressed in percentages. The right panel displays the distribution of bank's equity growth rates, expressed in percentages. Vertical axes are probabilities.

in the scale-free case. The median loan rate is the lowest in the complete network. The scale-free network observes a bimodal distribution of loan rates, where a small number of banks set the loan rate to the level which is lower than it is in the complete network case, and the rest set their loan rates at a relatively high level.

5.2 Network Shocks

Next, I consider network destruction shocks of different sizes.

5.2.1 100% Destruction of Complete Network

First, I analyze a removal of all the interbank connections in a complete network. I call this scenario an interbank market freeze. Figure 9 displays the impulse responses of economic variables. When banks cannot trade with each other, they always have to use discount window loans if they observe a reserve deficit. Thus, banks observe the highest possible cost of a liquidity deficit at the time of the shock. On impact, the aggregate loan rate increases and banks increase their holdings of cash assets relatively to loans. As banks experience a higher cost of liquidity funding and a drop in loan issuances, the aggregate equity in the banking sector decreases. Due to the decrease in loan supply, the firm's output contracts and household wages drop. As a result, the household provides less deposits to the banking system in the next period, which results in a further decline of aggregate lending and bank equity. Although banks initially choose to hold more cash relatively to loans, the consequent drop in deposits results in an overall reduction of the banking sector's portfolio.

Figure 10 displays the snapshots of the loan rate and the equity growth rate distributions at specific times after the shock. When the interbank network is destroyed, not only the distributional mean of the loan rates increases, but the variance of loan rates also goes up. Although the mean converges back to the initial level quite quickly, the variance in individual banks' loan rates persists for multiple periods. The distributional changes in Figure 10 are observed because of the "uneven" recovery of the interbank network. In particular, because only some banks get to rebuild their network connections early after the network shock, they get a competitive advantage over others and can accumulate equity faster.

5.2.2 Partial Destruction of a Complete Network

Figure 11 displays the responses of variables to a partial network destruction shock. When only some of the network connections are removed, the expected cost of a liquidity deficit does not change by as much as it does in the interbank market freeze scenario. As a result, the aggregate loan rate changes by less on impact. Interestingly, the direction of the aggregate

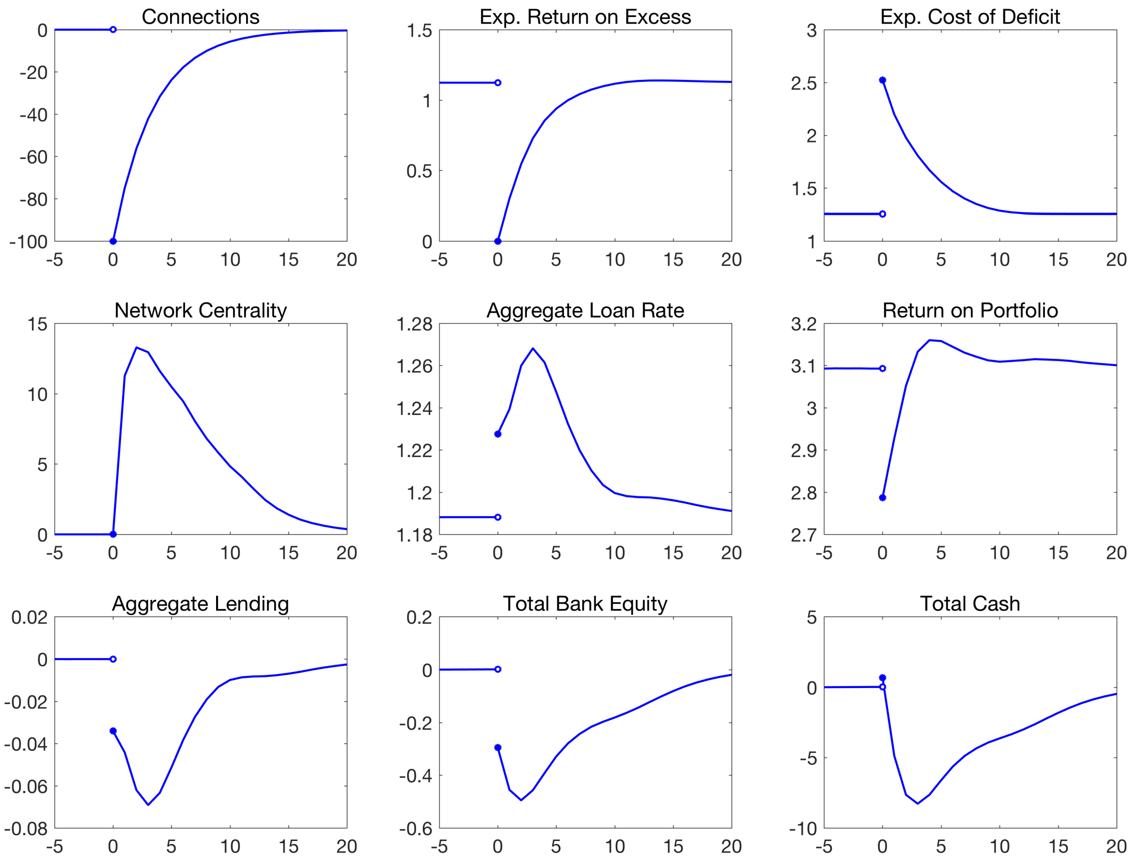


Figure 9. Complete Network Destruction Shock (100 simulations)

The figure displays impulse responses to removal of 100% links for a network with 100 banks. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations. Standard deviations σ are in relative terms.

loan rate's response changes as the size of the network shock decreases. In particular, there exists some threshold value, ζ^*, such that when less than ζ^* of connections are destroyed, the aggregate loan rate decreases on impact, as opposed to increasing in the case when more that ζ^* fraction of links are removed from the network. This is because assets allocation decisions are different between the two regimes. In particular, on average, banks increase their loan issuances and decrease cash holdings if $\zeta < \zeta^*$, whereas they do the opposite if $\zeta \geq \zeta^*$. In both cases, however, equity decreases, resulting in drop of portfolio size one period after the shock.

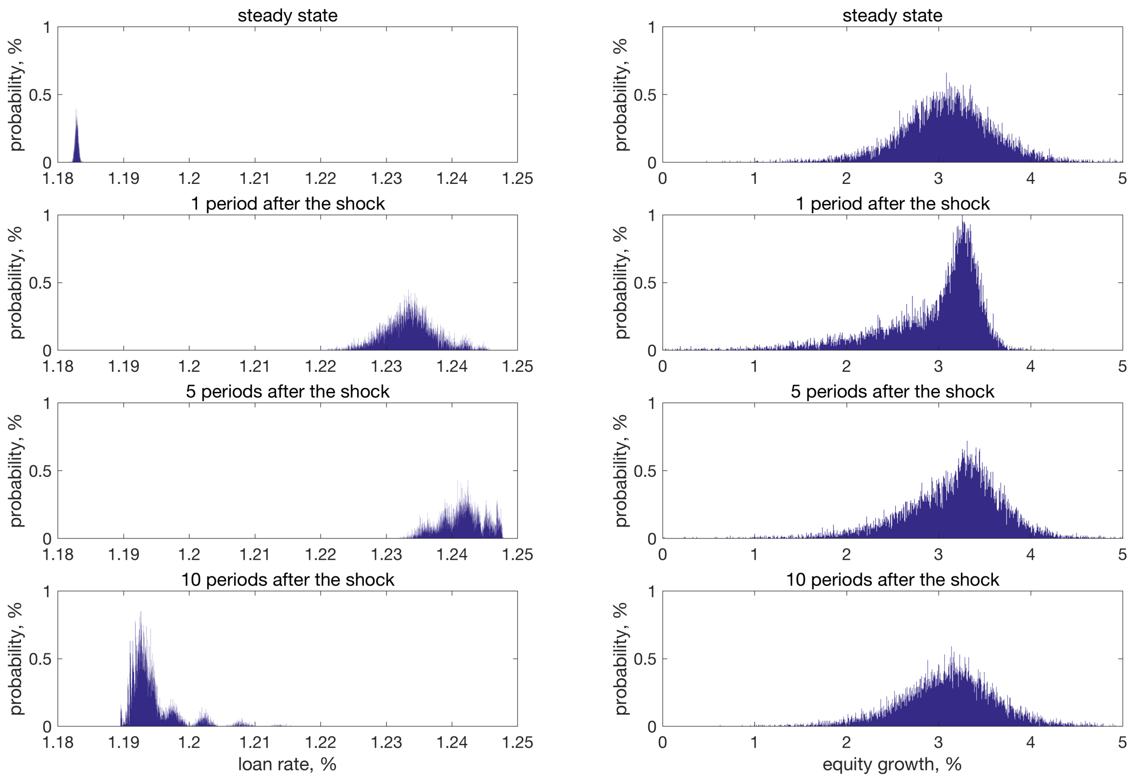


Figure 10. Loan Rates and Equity following the Network Shock

The figure displays distributions during particular times after the shock. The left panel shows the distribution of interest rates. Interest rates are expressed in percent. The right panel displays the distribution of bank's equity. Vertical axes are probabilities.

5.2.3 Shocks to Networks with Different Topologies

Next, I compare how the responses of variables to network disruption shocks in the complete network compare to the responses in the circle, random, and scale-free networks. The variables behave differently, depending on the initial density and topology of the incomplete network. Figures 12 and 13 display the results.

If the incomplete networks are initially 60 percent dense, the aggregate loan rate increases following the shock by a much smaller amount initially than it does in the complete network case. The effect on banks' equity is also smaller if the network is incomplete. However, the shock persists for much longer when the incomplete network is shocked compared to the destruction of the complete network. Note that when the incomplete networks are as

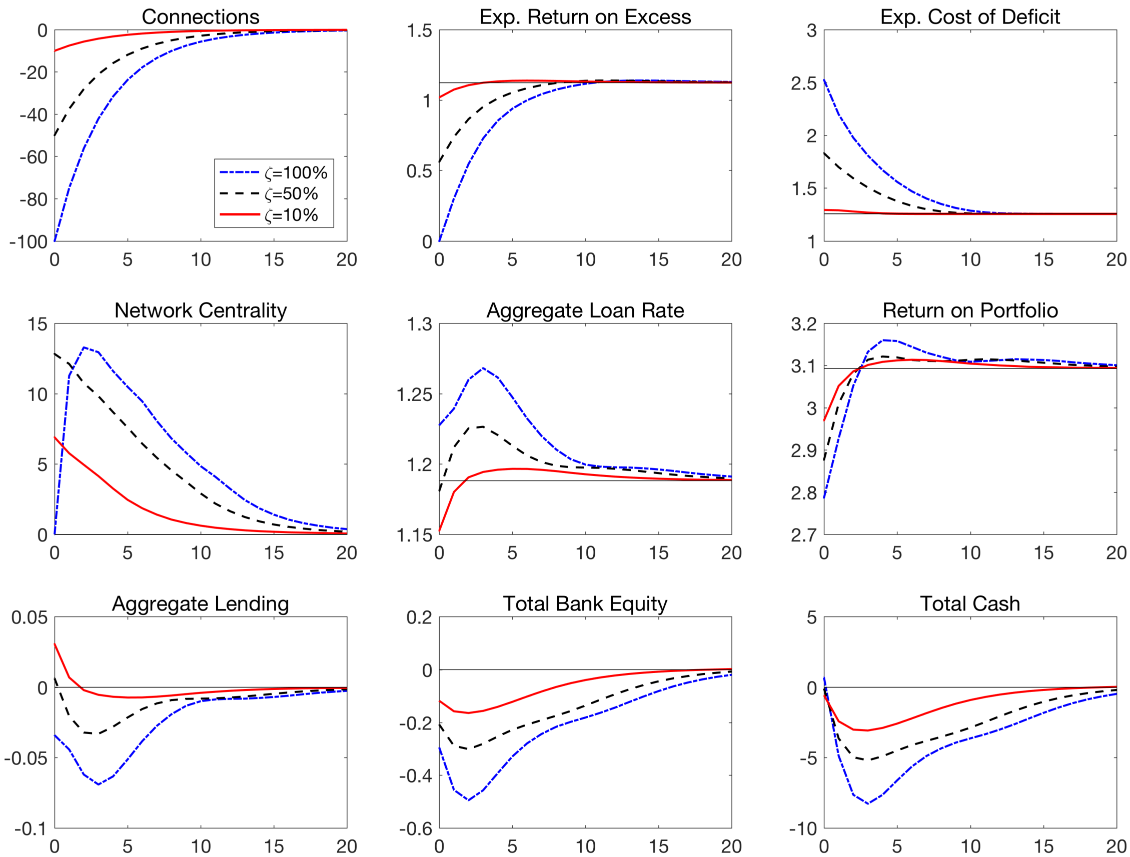


Figure 11. Partial Destruction of Complete Network

The figure displays impulse responses to removal of different percentage of links , ζ, in a network with 100 banks. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations. Standard deviations σ are in relative terms.

dense as 60 percent, the difference in responses between the different types of the incomplete networks are not as pronounced.

When I decrease the incomplete network density to 10 percent, the aggregate loan rate decreases following the destruction shock as opposed to increasing for the 60-percent dense networks. However, one period after the shock it increases above the steady-state level and then gradually decreases. In the scale-free and random network cases, the loan rate falls below the initial level once again three periods after the shock. Compare to the complete network case, the destruction shock to the 10-percent dense incomplete networks is not as

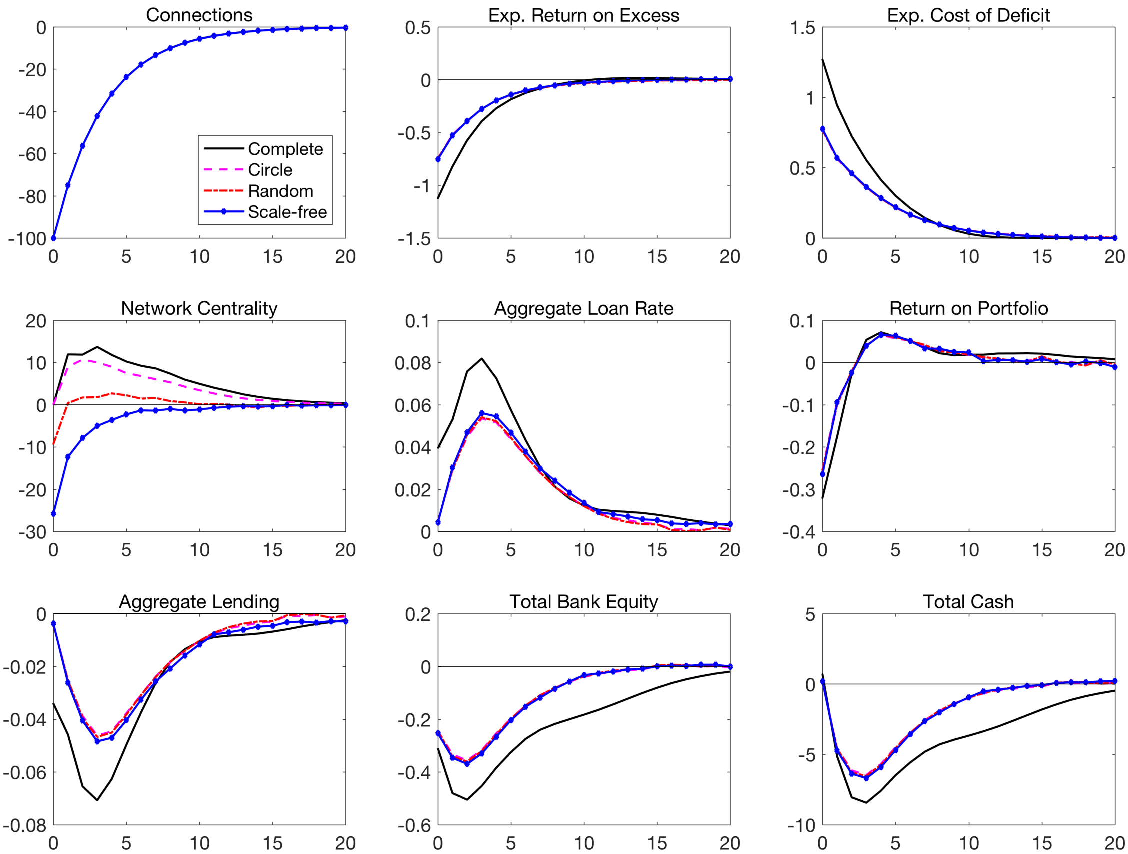


Figure 12. Interbank Market Freeze in Different Networks (60% dense)

The figure displays impulse responses to removal of all links for networks with different topologies. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

persistent. When it comes to the evolution of banks' equity, the scale-free and random networks observe an initial decline followed by a rise above the initial level two periods after the shock. This is not the case for the complete and circle networks – the aggregate equity remains below the initial level until the shock dissipates. I also consider a partial destruction of incomplete networks. I consider the case of 20-percent dense networks. The results are presented in Figure 14.

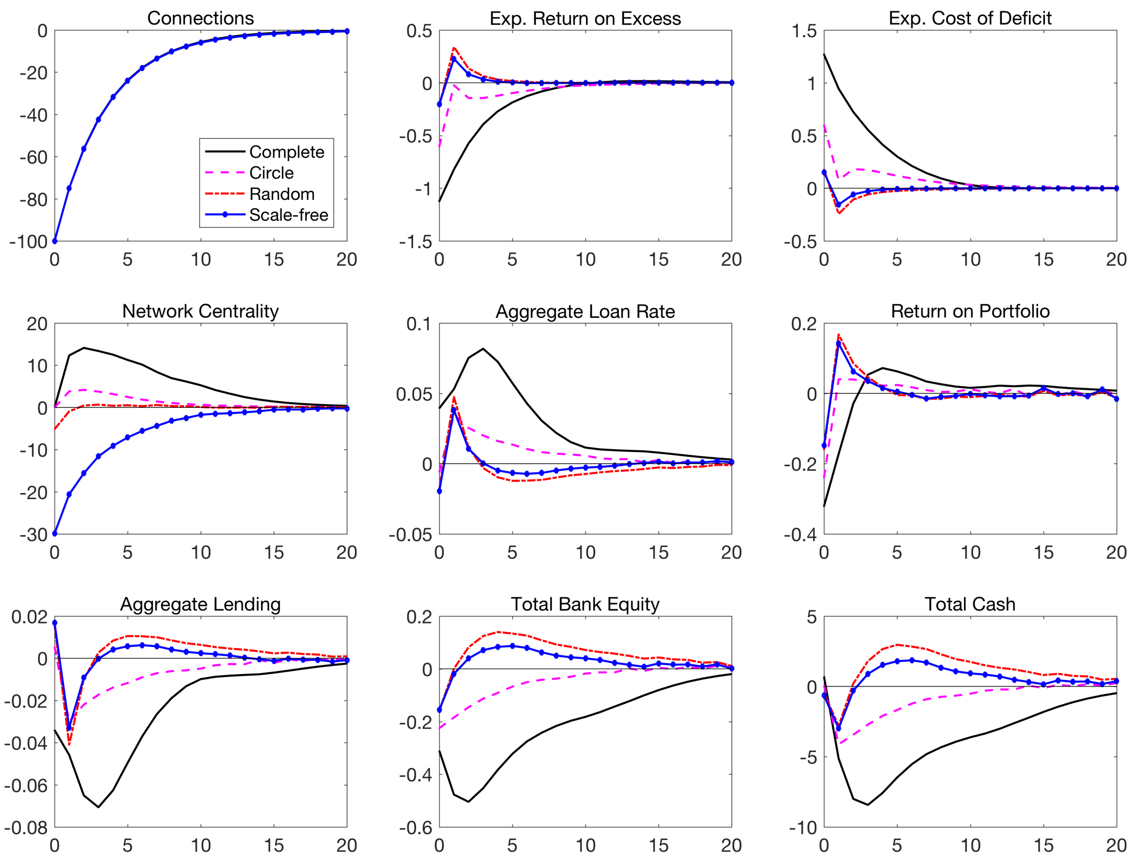


Figure 13. Interbank Market Freeze in Different Networks (10% dense)

The figure displays impulse responses to removal of all links for networks with different topologies. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

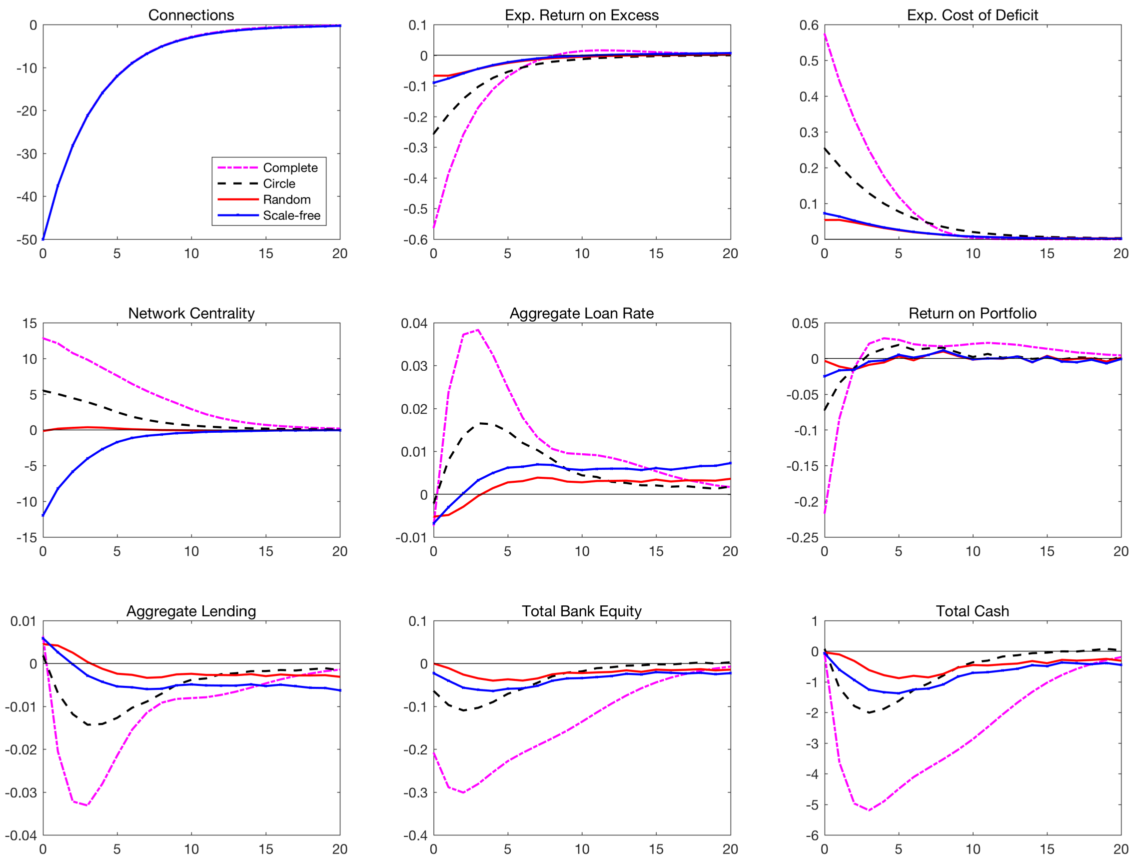


Figure 14. Partial Destruction of Networks with Different Topologies

The figure displays impulse responses to removal of half of the links in networks with different topologies. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

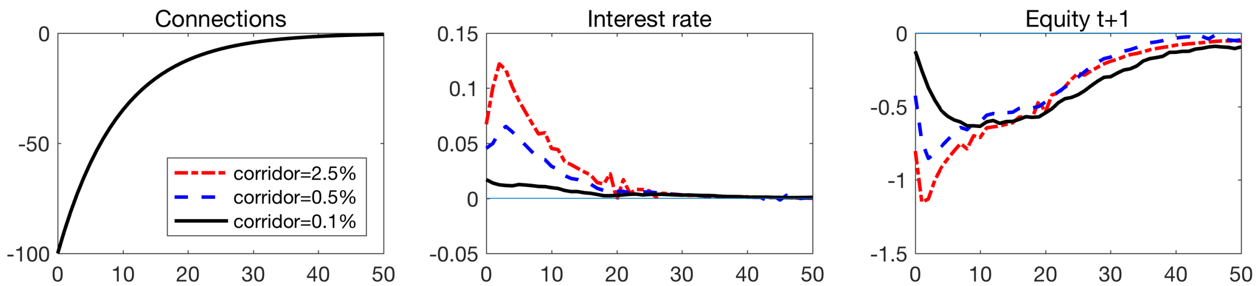


Figure 15. Interest Rate Corridor and Network Destruction Shock (complete network)

The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

6 Policy Implications

Next, I consider the interbank network destruction shock for different central bank policy rates. Recall that the central bank targets the interest rate on interbank loans, which is equal to $r_t^{FF} = \xi r_t^{ER} + (1-\xi) r_t^{DW}$. I conduct an experiment where the interest rate corridor, $r_t^{DW} - r_t^{ER}$, is changed such that r_t^{FF} is kept constant for all cases of the corridor. Figure 15 displays the responses of variables to a 100% network destruction shock for the complete interbank network. When the corridor is at 2.5% (relatively wide), banks' equity reduces by approximately 0.9 percent more at the time of the shock than it does in a narrow corridor case. However, the shock propagates (in the response of equity) for a longer time when the interest rate corridor is narrow.

I next check how the interest rate changes affect the economy for the different cases of the incomplete interbank network. Figures 16-18 display the results. The complete case outcomes hold for all the incomplete network cases.

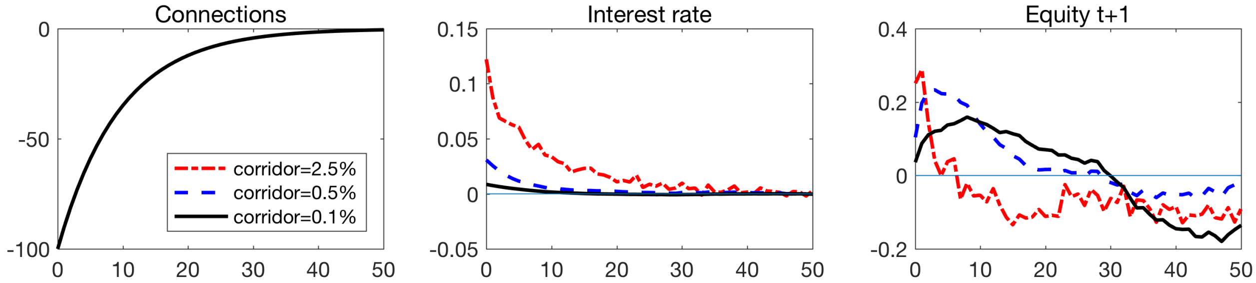


Figure 16. Interest Rate Corridor and Network Destruction Shock (circle network)

The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

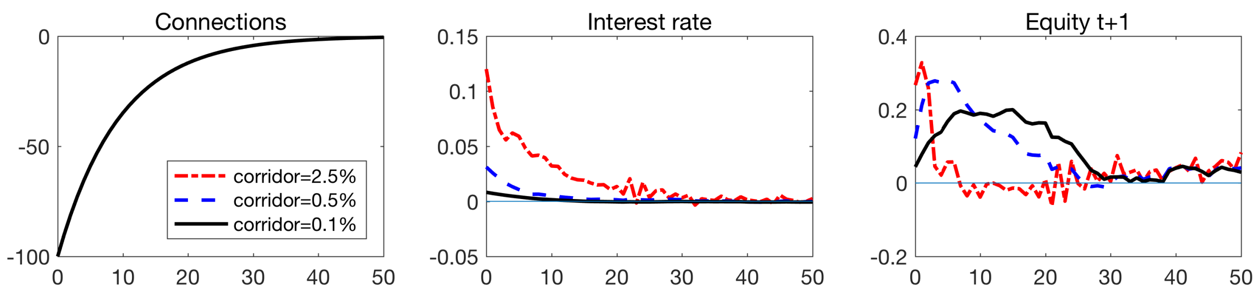


Figure 17. Interest Rate Corridor and Network Destruction Shock (random network)

The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

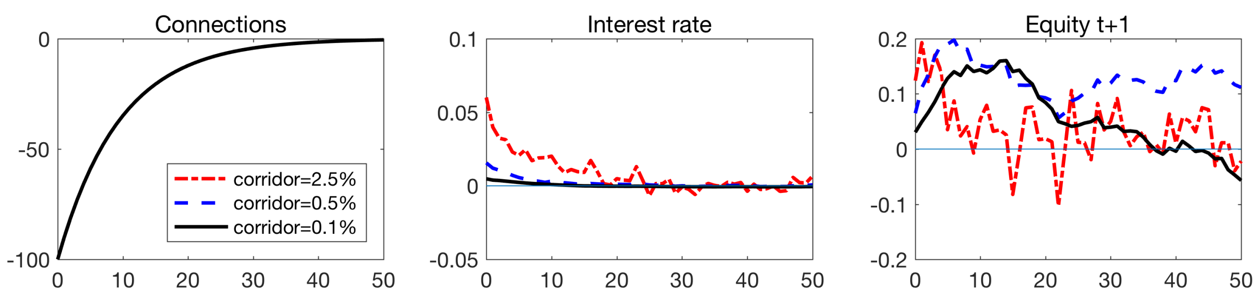


Figure 18. Interest Rate Corridor and Network Destruction Shock (scale-free network)

The figure displays impulse responses to removal of all links in the complete network for different interest rate corridors. Vertical axes are percent deviations from steady state for level variables and deviations from steady state in percentage points for interest rates and standard deviations.

7 Conclusion

This paper presents a dynamic macro model with an agent-based banking sector where banks are interconnected with each other via the interbank network. The quantitative exercises show that the structure and dynamics of the interbank network are essential for our understanding how the financial sector influences the real economy. In particular, the response of the economy to an interbank market freeze is qualitatively different from the response to a smaller interbank network disruption, which implies potentially different strategies for monetary policy. Depending on the central bank's policy, the shocks to the interbank network may matter more or less for aggregate lending. This presents a potential trade-off for monetary policy. Ongoing work includes further investigation of how central banks may be able to mitigate the distress in the financial sector. One other extension that is a subject of the ongoing research is embedding an endogenous mechanism of interbank network formation into the model.

References

Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1):1–33, February 2000. URL http://ideas.repec.org/a/ucp/jpolec/v108y2001i1p1-33.html.

Andrew G Atkeson, Andrea L Eisfeldt, and Pierre-Olivier Weill. The market for otc credit derivatives. In *UCLA Working Paper*. 2012.

A.L. Barabási. *Network Science*. Cambridge University Press, 2016. ISBN 9781107076266. URL https://books.google.com/books?id=iLtGDQAAQBAJ.

Albert-László Barabási and Réka Albert. Emergence of scaling in random networks. *science*, 286(5439):509–512, 1999.

Morten Bech and Cyril Monnet. Sorting in the interbank money market. Technical report, Citeseer, 2014.

Morten L Bech and Enghin Atalay. The topology of the federal funds market. *Physica A: Statistical Mechanics and its Applications*, 389(22):5223–5246, 2010.

Daniel O Beltran, Valentin Bolotnyy, and Elizabeth Klee. Un-networking: The evolution of networks in the federal funds market. 2015.

Javier Bianchi and Saki Bigio. Banks, liquidity management and monetary policy. NBER Working Papers 20490, National Bureau of Economic Research, Inc, September 2014. URL http://ideas.repec.org/p/nbr/nberwo/20490.html.

Francisco Blasques, Falk Brauning, and Iman Van Lelyveld. A dynamic network model of the unsecured interbank lending market. Working Papers 16-3, Federal Reserve Bank of Boston, April 2016. URL https://ideas.repec.org/p/fip/fedbwp/16-3.html.

Michael Boss, Helmut Elsinger, Martin Summer, and Stefan Thurner 4. Network topology of the interbank market. *Quantitative Finance*, 4(6):677–684, 2004.

Falk Bräuning, Falko Fecht, et al. *Relationship lending in the interbank market and the price of liquidity*. Deutsche Bundesbank, 2012.

Charles W Calomiris and Mark A Carlson. Interbank networks in the national banking era: their purpose and their role in the panic of 1893. 2016.

Ben Craig and Goetz von Peter. Interbank tiering and money center banks. *Journal of Financial Intermediation*, 23(3):322–347, 2014. URL http://ideas.repec.org/a/eee/jfinin/v23y2014i3p322-347.html.

Paul Erdös and Alfréd Rényi. On random graphs, i. *Publicationes Mathematicae (Debrecen)*, 6:290–297, 1959.

Xavier Freixas, Bruno Parigi, and Jean Charles Rochet. Systemic risk, interbank relations and liquidity provision by the central bank. Economics Working Papers 440, Department of Economics and Business, Universitat Pompeu Fabra, October 1998. URL http://ideas.repec.org/p/upf/upfgen/440.html.

Agam Gupta, Molly M King, James Magdanz, Regina Martinez, Matteo Smerlak, and Brady Stoll. Critical connectivity in banking networks. *SFI CSSS. SFI CSSS report*, 2013.

Hajime Inaoka, Takuto Ninomiya, Ken Taniguchi, Tokiko Shimizu, Hideki Takayasu, et al. Fractal network derived from banking transaction–an analysis of network structures formed by financial institutions. 2004.

Giulia Iori, Giulia De Masi, Ovidiu Vasile Precup, Giampaolo Gabbi, and Guido Caldarelli. A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278, 2008.

Konstantin Klemm and Victor M Eguiluz. Growing scale-free networks with small-world behavior. *Physical Review E*, 65(5):057102, 2002.

Simone Lenzu and Gabriele Tedeschi. Systemic risk on different interbank network topologies. Working Papers 375, Universita' Politecnica delle Marche (I), Dipartimento di Scienze Economiche e Sociali, April 2012. URL http://ideas.repec.org/p/anc/wpaper/375.html.

Sergei Maslov and Kim Sneppen. Specificity and stability in topology of protein networks. *Science*, 296(5569):910–913, 2002. ISSN 0036-8075. doi: 10.1126/science.1065103. URL http://science.sciencemag.org/content/296/5569/910.

Erlend Nier, Jing Yang, Tanju Yorulmazer, and Amadeo Alentorn. Network models and financial stability. *Journal of Economic Dynamics and Control*, 31(6):2033–2060, 2007.

Kimmo Soramäki, Morten L Bech, Jeffrey Arnold, Robert J Glass, and Walter E Beyeler. The topology of interbank payment flows. *Physica A: Statistical Mechanics and its Applications*, 379(1):317–333, 2007.

Anne Vila, Kimmo Soramaki, and Peter Zimmerman. The sterling unsecured loan market during 2006-08: insights from network theory. 2010.

Duncan J. Watts and Steven H. Strogatz. Collective dynamics of /'small-world/' networks. *Nature*, 393(6684):440–442, 06 1998. URL http://dx.doi.org/10.1038/30918.

A Proofs

A.1 Derivation of the Equation for the Total Expected Cost of a Reserve Deficit

Taking an expectation over bank's deficit of cash assets:

$$\begin{aligned}\mathbb{E}_\omega r_{it}^x = & \chi_{it}^L \int_{-\infty}^{\omega_{it}^*} [(\rho_t + (1-\rho_t)\omega) D_{it} - C_{it}] f(\omega) d\omega \\ & + \chi_{it}^B \int_{\omega_{it}^*}^{1} [(\rho_t + (1-\rho_t)\omega) D_{it} - C_{it}] f(\omega) d\omega\end{aligned} \tag{60}$$

Using the definition for the threshold value of the withdrawal shock:

$$\omega_{it}^* = \left(\frac{C_{it}}{D_{it}} - \rho_t\right) / (1 - \rho_t),$$

the bank's deficit can be rewritten as:

$$X_{it} \equiv (\rho_t + (1-\rho_t)\,\omega_{it}) D_{it} - C_{it} \equiv (1-\rho_t) D_{it} (\omega_{it} - \omega_{it}^*)$$

Substituting the above in (60):

$$\mathbb{E}_\omega r_{it}^x = (1-\rho_t) D_{it} \left[\chi_{it}^L \int_{-\infty}^{\omega_{it}^*} (\omega - \omega_{it}^*) f(\omega) d\omega + \chi_{it}^B \int_{\omega_{it}^*}^{1} (\omega - \omega_{it}^*) f(\omega) d\omega \right] \tag{61}$$

Rewriting the second integral:

$$\begin{aligned}\int_{\omega_{it}^*}^{1} (\omega - \omega_{it}^*) f(\omega) d\omega &= \int_{-\infty}^{1} (\omega - \omega_{it}^*) f(\omega) d\omega - \int_{-\infty}^{\omega_{it}^*} (\omega - \omega_{it}^*) f(\omega) d\omega \\ &= \bar{\omega} - \omega_{it}^* - \int_{-\infty}^{\omega_{it}^*} (\omega - \omega_{it}^*) f(\omega) d\omega\end{aligned}$$

where $\bar{\omega}$ is the mean of withdrawal shocks. Substituting in (61):

$$\begin{aligned}
\mathbb{E}_\omega r^x_{it} &= (1-\rho_t) D_{it}\left[\chi^L_{it}\int_{-\infty}^{\omega^*_{it}}(\omega-\omega^*_{it})f(\omega)d\omega+\chi^B_{it}\left(\bar{\omega}-\omega^*_{it}-\int_{-\infty}^{\omega^*_{it}}(\omega-\omega^*_{it})f(\omega)d\omega\right)\right] \\
&= (1-\rho_t) D_{it}\left[\left(\chi^L_{it}-\chi^B_{it}\right)\int_{-\infty}^{\omega^*_{it}}(\omega-\omega^*_{it})f(\omega)d\omega+\chi^B_{it}\left(\bar{\omega}-\omega^*_{it}\right)\right] \\
&= (1-\rho_t) D_{it}\left[\chi^B_{it}\left(\bar{\omega}-\omega^*_{it}\right)+\left(\chi^B_{it}-\chi^L_{it}\right)\left(\omega^*_{it}F\left(\omega^*_{it}\right)-\int_{-\infty}^{\omega^*_{it}}\omega f(\omega)d\omega\right)\right]
\end{aligned}$$

A.2 Derivation of the Loan Supply Equation

Recall the optimality condition (25):

$$R^b_{it} = -\frac{\partial \mathbb{E}_\omega r^x_{it}\left(1, L_{it}\right)}{\partial L_{it}}$$

where

$$\mathbb{E}_\omega r^x_{it}\left(1, L_{it}\right) = \frac{\mathbb{E}_\omega r^x_{it}\left(D_{it}, C_{it}\right)}{D_{it}}$$

Differentiating with respect to L_{it}:

$$\begin{aligned}
\frac{\partial \mathbb{E}_\omega r^x_{it}\left(1, L_{it}\right)}{\partial L_{it}} &= -\chi^B_{it}+\left(\chi^B_{it}-\chi^L_{it}\right)\left(F\left(\tfrac{L_{it}-\rho_t}{1-\rho_t}\right)+\cancel{\frac{L_{it}-\rho_t}{1-\rho_t}f\left(\tfrac{L_{it}-\rho_t}{1-\rho_t}\right)}-\cancel{\frac{L_{it}-\rho_t}{1-\rho_t}f\left(\tfrac{L_{it}-\rho_t}{1-\rho_t}\right)}\right) \\
&= -\chi^B_{it}+\left(\chi^B_{it}-\chi^L_{it}\right)F\left(\frac{L_{it}-\rho_t}{1-\rho_t}\right)
\end{aligned}$$

Substituting in (25):

$$R^b_{it} = \chi^B_{it}-\left(\chi^B_{it}-\chi^L_{it}\right)F\left(\frac{L_{it}-\rho_t}{1-\rho_t}\right)$$

B Matching Probabilities

Banks have to make portfolio decisions before the deposit shocks are realized. I define a vector of bank deficits:

$$X_{it} = \omega_{it} D_{it} - C_{it} \tag{62}$$

where a negative value implies that a bank has a surplus.

Let ω_{it}^* be the value of deposit withdrawal that makes deficit X_{it} equal 0:

$$\omega_{it}^* = \frac{C_{it}}{D_{it}} = \frac{c_{it}}{d_{it}}$$

Then if $\omega_{it} \leq \omega_{it}^*$, there is a surplus $X_{it} \leq 0$ and i is a lender, and if $\omega_{it} > \omega_{it}^*$, there is a deficit $X_{it} > 0$ and i is a borrower. Recall

$$p_{it}^{BL}(G) = \min\left[1,\ \frac{\Upsilon_i^+(G_i)}{\Upsilon_i^-(G)}\right]$$

where $K = \sum_j\ G_{jk} \cdot \mathbb{1}_{\{G_{ijt}=1\}}$.

$$\begin{aligned}
\Upsilon_i^+ &= \sum_j G_{ijt} \cdot F\left(\omega_j \leq \omega_j^*\right) \mathbb{E}\left[X_{jt}(\omega_j) \mid \omega_j \leq \omega_j^*\right] \\
&= \sum_j G_{ijt} \cdot F\left(\omega_j \leq \omega_j^*\right) \int_{-\infty}^{\omega_j^*} \left(\omega \tilde{D}_j - \tilde{C}_j\right) f(\omega) d\omega \\
&= \sum_j G_{ijt} \cdot F\left(\omega_j \leq \omega_j^*\right) (1-\theta) E_j \int_{-\infty}^{\omega_j^*} \left(\omega \tilde{d}_j - \tilde{c}_j\right) f(\omega) d\omega \\
&= (1-\theta) \sum_j G_{ijt} \cdot F\left(\omega_j \leq \omega_j^*\right) E_j \int_{-\infty}^{\omega_j^*} \left(\omega \tilde{d}_j - \tilde{c}_j\right) f(\omega) d\omega
\end{aligned}$$

The integral $\int_{-\infty}^{\omega_j^*} \left(\omega \tilde{d}_j - \tilde{c}_j\right) f(\omega) d\omega$ is the same for all j, thus it can be taken out of the sum:

$$\Upsilon_i^+ = (1-\theta)\, F\left(\omega_j \leq \omega_j^*\right) \int_{-\infty}^{\omega^*} \left(\omega \tilde{d} - \tilde{c}\right) f(\omega) d\omega \cdot \sum_j G_{ijt} \cdot E_j$$
$$= (1-\theta)\, F\left(\omega_j \leq \omega_j^*\right) \mathbb{E}\left[x(\omega) \mid \omega \leq \omega^*\right] \cdot \sum_j G_{ijt} \cdot E_j$$

Equivalently,

$$\Upsilon_i^- = \sum_k \ \mathbb{1}_{\{K_k \geq 1\}} \cdot F\left(\omega_j > \omega_j^*\right) \mathbb{E}\left[X_{kt}(\omega_k) \mid \omega_k > \omega_k^*\right]$$
$$= (1-\theta)\, \mathbb{E}\left[x(\omega) \mid \omega > \omega^*\right] \cdot F\left(\omega_j > \omega_j^*\right) \sum_k \ \mathbb{1}_{\{K_k \geq 1\}} E_k$$

$$p_{it}^{BL}(G) = \min\left[1,\ \frac{(1-\theta)\, F(\omega \leq \omega^*)\, \mathbb{E}\left[x(\omega) \mid \omega \leq \omega^*\right] \cdot \sum_j G_{ijt} \cdot E_j}{(1-\theta)\, F(\omega > \omega^*)\, \mathbb{E}\left[x(\omega) \mid \omega > \omega^*\right] \cdot \sum_k \ \mathbb{1}_{\{K_k \geq 1\}} E_k}\right]$$
$$= \min\left[1,\ \frac{F(\omega \leq \omega^*)\, \mathbb{E}\left[x(\omega) \mid \omega \leq \omega^*\right] \cdot \sum_j G_{ijt} \cdot E_j}{F(\omega > \omega^*)\, \mathbb{E}\left[x(\omega) \mid \omega > \omega^*\right] \cdot \sum_k \ \mathbb{1}_{\{K_k \geq 1\}} \cdot E_k}\right]$$

$$p_{it}^{BL}(G) = \min\left[1,\ \frac{F(\omega \leq \omega^*)\, \mathbb{E}\left[x(\omega) \mid \omega \leq \omega^*\right]}{F(\omega > \omega^*)\, \mathbb{E}\left[x(\omega) \mid \omega > \omega^*\right]} \cdot \frac{\sum_j G_{ijt} \cdot E_j}{\sum_k \ \mathbb{1}_{\{K_k \geq 1\}} \cdot E_k}\right] \tag{63}$$

where $K = \sum_j \ G_{jk} \cdot \mathbb{1}_{\{G_{ijt}=1\}}$.

Similar procedure results in an expression for the probability of lending order matching with a borrowing order. Recall that:

$$p_{it}^{LB}(G) = \min\left[1,\ \frac{\Gamma_i^-}{\Gamma_i^+}\right]$$

where Γ_i^- is the mass of reserve deficits for i's neighbors and Γ_i^+ is the mass of lending orders available to i's neighbors with borrowing orders.

$$p_{it}^{LB}\left(G\right)=\min\left[1,\ \frac{F\left(\omega>\omega^{*}\right)\mathbb{E}\left[x(\omega)\mid\omega>\omega^{*}\right]}{F\left(\omega\leq\omega^{*}\right)\mathbb{E}\left[x(\omega)\mid\omega\leq\omega^{*}\right]}\cdot\frac{\sum_{j}G_{ijt}\cdot E_{j}}{\sum_{k}\ \mathbb{1}_{\{K_{k}\geq1\}}\cdot E_{k}}\right] \tag{64}$$

I separate the components of the probabilities into common and idiosyncratic:

$$\begin{aligned} \text{common: }\psi &= \frac{F\left(\omega>\omega^{*}\right)\mathbb{E}\left[x(\omega)\mid\omega>\omega^{*}\right]}{F\left(\omega\leq\omega^{*}\right)\mathbb{E}\left[x(\omega)\mid\omega\leq\omega^{*}\right]} \\ \text{idiosyncratic: }\Psi_{i}\left(G_{i}\right) &= \frac{\sum_{j}G_{ijt}\cdot E_{j}}{\sum_{k}\ \mathbb{1}_{\{K_{k}\geq1\}}\cdot E_{k}} \end{aligned}$$

Then,

$$p_{it}^{BL}\left(G\right)=\min\left[1,\frac{1}{\psi}\Psi_{i}\left(G_{i}\right)\right] \tag{65}$$

$$p_{it}^{LB}\left(G\right)=\min\left[1,\ \psi\ \Psi_{i}\left(G_{i}\right)\right] \tag{66}$$

C Generalized Bank's Problem

This section closely follows the derivation of the bank's problem in Bianchi and Bigio (2014). Banks maximize their expected lifetime utility:

$$\max_{D_{it},B_{it},C_{it},DIV_{i}} \qquad \mathbb{E}_{0}\sum_{t\geq0}\left(\beta\zeta\right)^{t}\ \frac{DIV^{1-\gamma}}{1-\gamma}$$

$$\text{s.t.} \qquad E_{it}=B_{it}+C_{it}+DIV_{i}-D_{it} \tag{67}$$

$$E_{it}^{\prime}=R_{it}^{b}B_{it}-R^{d}D_{it}+r_{it}^{x}C_{it}-\omega_{it}\left(r_{it}^{x}-R^{d}\right)D_{it} \tag{68}$$

$$D_{it}\leq\kappa\left(B_{it}+C_{it}-D_{it}\right) \tag{69}$$

$$B_{it},C_{it},D_{it}\geq0$$

I denote the aggregate state by Z and solve the above problem by the method of dynamic programming. Vector $Z=\left\{r_{t}^{DW};r_{t}^{ER};F\left(\omega\right);G\right\}$ summarizes the aggregate state, which includes policy rates, distribution of withdrawal shocks, and the network matrix. Rewriting the problem:

$$V\left(E_{it}, Z\right) = \max_{D_{it}, B_{it}, C_{it}, DIV_i} \frac{DIV^{1-\gamma}}{1-\gamma} + \beta\zeta\mathbb{E}\left[V\left(E'_{it}, Z'\right)\right] \tag{70}$$

$$\begin{aligned}
\text{s.t.} \qquad & E_{it} = B_{it} + C_{it} + DIV_i - D_{it} \\
& E'_{it} = R^b_{it} B_{it} - R^d D_{it} + r^x_{it} C_{it} - \omega_{it}\left(r^x_{it} - R^d\right) D_{it} \\
& D_{it} \leq \kappa\left(B_{it} + C_{it} - D_{it}\right) \\
& B_{it}, C_{it}, D_{it} \geq 0
\end{aligned}$$

C.1 Homogeneity

I define a fraction of equity that a bank allocates towards dividends as $div_i \equiv DIV_i/E_{it}$. The utility function can be written as:

$$U\left(DIV_i\right) = E_{it}^{1-\gamma} \cdot U\left(div_i\right)$$

I guess that the value function satisfies:

$$V\left(E_{it}, Z\right) = v\left(Z\right) E_{it}^{1-\gamma}$$

where $v\left(Z\right)$ is the slope of the value function. The value function (70) can be rewritten as:

$$V\left(E_{it}, Z\right) = E_{it}^{1-\gamma}\left[\max_{D_{it}, B_{it}, C_{it}, div_i} \frac{{div_i}^{1-\gamma}}{1-\gamma} + \beta\zeta\mathbb{E}_\omega\mathbb{E}\, v\left(Z'|Z\right)\left[\frac{E'_{it}}{E_{it}}\right]^{1-\gamma}\right]$$

Consider the budget constraint (67). Dividing it by E_{it} results in:

$$1 = b_{it} + c_{it} + div_i - d_{it} \tag{71}$$

where deposits, loans, and reserves are expressed as fractions of equity:

$$[d_{it} \;\; b_{it} \;\; c_{it}] \equiv \left[\frac{D_{it}}{E_{it}} \;\; \frac{B_{it}}{E_{it}} \;\; \frac{C_{it}}{E_{it}}\right] \tag{72}$$

The level of equity in the beginning of a period is non-negative, thus, dividing the capital requirement by E_{it} results in:

$$d_{it} \leq \kappa \left(b_{it} + c_{it} - d_{it}\right) \tag{73}$$

Consider the evolution of equity (68). All the terms on the right-hand side are linear in equity. Dividing the equation by E_{it} yields:

$$\frac{E'_{it}}{E_{it}} = R^b_{it} b_{it} - R^d d_{it} + r^x_{it} c_{it} - \omega_{it}\left(r^x_{it} - R^d\right) d_{it} \tag{74}$$

where $\frac{E'_{it}}{E_{it}}$ is equity growth between two consecutive periods, which is equal to the sum of the realized returns on loans and reserves net of the cost of deposits.

Problem 5 *The scale-invariant problem of a bank is:*

$$v\left(Z\right) = \max_{d_{it}, b_{it}, c_{it}, div_i} \frac{{div_i}^{1-\gamma}}{1-\gamma} + \beta\zeta \mathbb{E}_\omega \mathbb{E}\, v\left(Z'|Z\right) \left[\frac{E'_{it}}{E_{it}}\right]^{1-\gamma} \tag{75}$$

$$\begin{aligned} s.t. \qquad & 1 = b_{it} + c_{it} + div_i - d_{it} \\ & \frac{E'_{it}}{E_{it}} = R^b_{it} b_{it} - R^d d_{it} + r^x_{it} c_{it} - \omega_{it}\left(r^x_{it} - R^d\right) d_{it} \\ & d_{it} \leq \kappa\left(b_{it} + c_{it} - d_{it}\right) \\ & b_{it}, c_{it}, d_{it} \geq 0 \end{aligned}$$

Policy rules that solve the original problem are equivalent to the policy rules that solve Problem 5 multiplied by equity.

C.2 Portfolio Separation

Rewrite the budget constraint (71) as follows:

$$1 - div_i = b_{it} + c_{it} - d_{it}$$

The left-hand side constitutes the fraction of equity that is split between investment in assets with different returns. These can be thought of as portfolio shares of three assets: loans, reserves, and deposits. I define these shares as:

$$\hat{b}_i = \frac{b_{it}}{1 - div_i}, \quad \hat{c}_i = \frac{c_{it}}{1 - div_i}, \quad \hat{d}_i = \frac{d_{it}}{1 - div_i} \tag{76}$$

Using the definitions above, the budget constraint and the capital requirement can be rewritten as:

$$1 = \hat{b}_i + \hat{c}_i - \hat{d}_i \tag{77}$$

$$\hat{d}_i \leq \frac{\kappa}{1+\kappa}\left(\hat{b}_i + \hat{c}_i\right) \tag{78}$$

respectively. Expressing the evolution of equity in terms of portfolio shares results in:

$$\frac{E'_{it}}{E_{it}} = (1 - div_i)\left[R^b_{it}\hat{b}_i - R^d\hat{d}_i + r^x_{it}\hat{c}_i - \omega_{it}\left(r^x_{it} - R^d\right)\hat{d}_i\right]$$

Substituting the budget constraint (77) into the above:

$$\frac{E'_{it}}{E_{it}} = (1 - div_i)\left[R^b_{it} + \left(r^x_{it} - R^b_{it}\right)\hat{c}_i + R^b_{it}\hat{d}_i - \omega_{it}\left(r^x_{it} - R^d\right)\hat{d}_i\right] \tag{79}$$

Substituting the budget constraint (77) into the capital requirement (78) yields:

$$\hat{d}_i \leq \kappa \tag{80}$$

Since div_i is known at $t+1$, the value function (75) can be rewritten as:

$$v\left(Z\right) = \max_{\hat{d}_i, \hat{c}_i, div_i} \quad \frac{{div_i}^{1-\gamma}}{1-\gamma} + \beta\zeta\left(1-div_i\right)^{1-\gamma}\mathbb{E}\, v\left(Z'|Z\right)\mathbb{E}_\omega\left[R_i^E\right]^{1-\gamma}$$

where R_i^E is the realized return on bank's portfolio defined as:

$$R_i^E \equiv R_{it}^b\hat{b}_i - R^d\hat{d}_i + r_{it}^x\hat{c}_i - \omega_{it}\left(r_{it}^x - R^d\right)\hat{d}_i \tag{81}$$

Moreover, $\hat{c}_i$ and $\hat{d}_i$ enter only in the continuation value, thus, their optimal values can be found independently from optimal dividend. Solving problem 5 is equivalent to solving the following problem:

Problem 6 *The value function $v\left(\cdot\right)$ solves:*

$$\begin{aligned}
v\left(Z\right) = \max_{div_i} \quad & \frac{{div_i}^{1-\gamma}}{1-\gamma} + \beta\zeta\left(1-div_i\right)^{1-\gamma}\mathbb{E}\, v\left(Z'|Z\right)\max_{\hat{d}_i,\hat{c}_i}\,\mathbb{E}_\omega\left[R_i^E\right]^{1-\gamma} \\
s.t. \quad & R_i^E = R_{it}^b\hat{b}_i - R^d\hat{d}_i + r_{it}^x\hat{c}_i - \omega_{it}\left(r_{it}^x - R^d\right)\hat{d}_i \\
& \hat{d}_i \leq \kappa \\
& 0 \leq \hat{c}_i, \hat{d}_i
\end{aligned}$$

Next I consider the portfolio maximization problem.

C.3 Portfolio Maximization Problem

$$\begin{aligned}
\max_{\hat{d}_i,\hat{c}_i} \quad & \mathbb{E}_\omega\left[R_{it}^b\hat{b}_i - R^d\hat{d}_i + r_{it}^x\hat{c}_i - \omega_{it}\left(r_{it}^x - R^d\right)\hat{d}_i\right]^{1-\gamma} \\
\text{s.t.} \quad & \hat{d}_i \leq \kappa \\
& 0 \quad \leq \hat{c}_i, \hat{d}_i
\end{aligned} \tag{82}$$

A non-standard feature of this problem is that r_{it}^x has a discontinuity at the point where bank's reserve deficit is zero. This occurs when $\omega_{it} = \frac{\hat{c}_i}{\hat{d}_i}$. Since $\omega_{it} \leq 1$, then it must be that $\frac{\hat{c}_i}{\hat{d}_i} \leq 1$, which rules out $\hat{d}_i = 0$ in equilibrium. If the realized shock is below $\frac{\hat{c}_i}{\hat{d}_i}$, then the bank has excess reserves, which can be sold at χ^L_{it}. If the realized shock is above $\frac{\hat{c}_i}{\hat{d}_i}$, then the bank has a reserve deficit and has to buy reserves at χ^B_{it}. The portfolio problem can be rewritten as follows:

$$\begin{aligned}
\max_{\hat{d}_i,\hat{c}_i} \quad & \int_{-\infty}^{\frac{c_{it}}{d_{it}}} \left[R^E_i\left(\chi^L_{it}\right)\right]^{1-\gamma} f(\omega)\, d\omega + \int_{\frac{c_{it}}{d_{it}}}^{1} \left[R^E_i\left(\chi^B_{it}\right)\right]^{1-\gamma} f(\omega)\, d\omega \\
\text{s.t.} \quad & \hat{d}_i \leq \kappa \\
& 0 \ \ \leq \hat{c}_i \\
\text{where} \quad & R^E_i\left(r^x_{it}\right) = R^b_{it} + \left(r^x_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + r^x_{it}\cdot\omega_{it}\right)\hat{d}_i
\end{aligned}$$

Rewriting the problem:

$$\begin{aligned}
\max_{\hat{d}_i,\hat{c}_i} \quad & \int_{-\infty}^{\frac{c_{it}}{d_{it}}} \left[R^b_{it} + \left(\chi^L_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \omega_{it}\chi^L_{it}\right)\hat{d}_i\right]^{1-\gamma} f(\omega)\, d\omega \\
& + \int_{\frac{c_{it}}{d_{it}}}^{1} \left[R^b_{it} + \left(\chi^B_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \omega_{it}\chi^B_{it}\right)\hat{d}_i\right]^{1-\gamma} f(\omega)\, d\omega \\
& + \mu_i\left(\kappa - \hat{d}_i\right) + \lambda^1_{it}\hat{c}_i + \lambda^2_{it}\hat{d}_i
\end{aligned}$$

Differentiating w.r.t. $\hat{c}_i$:

$$\begin{aligned}
0 = {} & (1-\gamma)\left(\chi^L_{it} - R^b_{it}\right)\int_{-\infty}^{\frac{c_{it}}{d_{it}}} \left[R^b_{it} + \left(\chi^L_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \omega_{it}\chi^L_{it}\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
& + \frac{1}{\hat{d}_i}\left[R^b_{it} + \left(\cancel{\chi^L_{it}} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \cancel{\frac{c_{it}}{d_{it}}\chi^L_{it}}\right)\hat{d}_i\right]^{1-\gamma} \\
& + (1-\gamma)\left(\chi^B_{it} - R^b_{it}\right)\int_{\frac{c_{it}}{d_{it}}}^{1} \left[R^b_{it} + \left(\chi^B_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \omega_{it}\chi^B_{it}\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
& - \frac{1}{\hat{d}_i}\left[R^b_{it} + \left(\cancel{\chi^B_{it}} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \cancel{\frac{c_{it}}{d_{it}}\chi^B_{it}}\right)\hat{d}_i\right]^{1-\gamma} + \lambda^1_{it}
\end{aligned}$$

Simplyfying and dividing by $1-\gamma$:

$$-\frac{\lambda_{it}^1}{1-\gamma} = \left(\chi_{it}^L - R_{it}^b\right) \int_{-\infty}^{\frac{c_{it}}{d_{it}}} \left[R_{it}^b + \left(\chi_{it}^L - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^L\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega$$
$$+ \left(\chi_{it}^B - R_{it}^b\right) \int_{\frac{c_{it}}{d_{it}}}^{1} \left[R_{it}^b + \left(\chi_{it}^B - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^B\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega$$

Isolate R_{it}^b:

$$\begin{aligned}
-\frac{\lambda_{it}^1}{1-\gamma} = &\int_{-\infty}^{\frac{c_{it}}{d_{it}}} \chi_{it}^L \left[R_{it}^b + \left(\chi_{it}^L - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^L\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
&- R_{it}^b \int_{-\infty}^{\frac{c_{it}}{d_{it}}} \left[R_{it}^b + \left(\chi_{it}^L - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^L\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
&+ \int_{\frac{c_{it}}{d_{it}}}^{1} \chi_{it}^B \left[R_{it}^b + \left(\chi_{it}^B - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^B\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
&- R_{it}^b \int_{\frac{c_{it}}{d_{it}}}^{1} \left[R_{it}^b + \left(\chi_{it}^B - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^B\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega
\end{aligned}$$

Using the definition $R_i^E \equiv R_{it}^b + \left(\chi_{it}^L - R_{it}^b\right)\hat{c}_i - \left(R^d - R_{it}^b + \omega_{it}\chi_{it}^L\right)\hat{d}_i$, the above equation can be rewritten as follows:

$$\mathbb{E}_\omega \left[r_{it}^x \left[R_i^E\right]^{-\gamma}\right] - R_{it}^b \mathbb{E}_\omega \left[R_i^E\right]^{-\gamma} + \frac{\lambda_{it}^1}{1-\gamma} = 0$$

Applying formula for expectation of a product of two dependent variables:

$$\mathbb{E}_\omega r_{it}^x \cdot \mathbb{E}_\omega \left[R_i^E\right]^{-\gamma} + \mathbb{COV}\left\{r_{it}^x, \left[R_i^E\right]^{-\gamma}\right\} - R_{it}^b\, \mathbb{E}_\omega \left[R_i^E\right]^{-\gamma} + \frac{\lambda_{it}^1}{1-\gamma} = 0$$

Dividing by $\mathbb{E}_\omega \left[R_i^E\right]^{-\gamma}$:

$$R_{it}^b = \mathbb{E}_\omega r_{it}^x + \frac{\mathbb{COV}\left\{r_{it}^x, \left[R_i^E\right]^{-\gamma}\right\}}{\mathbb{E}_\omega \left[R_i^E\right]^{-\gamma}} + \frac{\lambda_{it}^1}{(1-\gamma)\mathbb{E}_\omega \left[R_i^E\right]^{-\gamma}}$$

where

$$\begin{aligned}
\mathbb{E}_\omega r^x_{it} &= \chi^L_{it} \int_{-\infty}^{\frac{c_{it}}{d_{it}}} f(\omega)\, d\omega + \chi^B_{it} \int_{\frac{c_{it}}{d_{it}}}^{1} f(\omega)\, d\omega \\
&= \chi^L_{it} \cdot F\left[\frac{c_{it}}{d_{it}}\right] + \chi^B_{it} \left(1 - F\left[\frac{c_{it}}{d_{it}}\right]\right) \\
&= \left(\chi^L_{it} - \chi^B_{it}\right) F\left[\frac{c_{it}}{d_{it}}\right] + \chi^B_{it}
\end{aligned}$$

Differentiating the objective w.r.t. $\hat{d}_i$:

$$\begin{aligned}
&- \left(R^d - R^b_{it} + \omega_{it}\chi^L_{it}\right) \int_{-\infty}^{\frac{c_{it}}{d_{it}}} \left[R^b_{it} + \left(\chi^L_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \omega_{it}\chi^L_{it}\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
&- \left(R^d - R^b_{it} + \omega_{it}\chi^B_{it}\right) \int_{\frac{c_{it}}{d_{it}}}^{1} \left[R^b_{it} + \left(\chi^B_{it} - R^b_{it}\right)\hat{c}_i - \left(R^d - R^b_{it} + \omega_{it}\chi^B_{it}\right)\hat{d}_i\right]^{-\gamma} f(\omega)\, d\omega \\
&- \frac{\mu_i}{1-\gamma} + \frac{\lambda^2_{it}}{1-\gamma} = 0
\end{aligned}$$

Rewriting in terms of R^E_i:

$$- \left(R^d - R^b_{it}\right)\mathbb{E}_\omega \left[R^E_i\right]^{-\gamma} - \mathbb{E}_\omega\left[\omega_{it} r^x_{it} \left[R^E_i\right]^{-\gamma}\right] + \frac{\lambda^2_{it} - \mu_i}{1-\gamma} = 0$$

Applying formula for expectation of a product of two dependent variables:

$$-\left(R^d - R^b_{it}\right)\mathbb{E}_\omega \left[R^E_i\right]^{-\gamma} - \mathbb{E}_\omega\left[\omega_{it} r^x_{it}\right]\mathbb{E}_\omega\left[R^E_i\right]^{-\gamma} - \mathbb{COV}\left\{\omega_{it} r^x_{it}, \left[R^E_i\right]^{-\gamma}\right\} + \frac{\lambda^2_{it} - \mu_i}{1-\gamma} = 0$$

Dividing by $\mathbb{E}_\omega\left[R^E_i\right]^{-\gamma}$:

$$R^d - R^b_{it} = -\mathbb{E}_\omega\left[\omega_{it} r^x_{it}\right] - \frac{\mathbb{COV}\left\{\omega_{it} r^x_{it}, \left[R^E_i\right]^{-\gamma}\right\}}{\mathbb{E}_\omega\left[R^E_i\right]^{-\gamma}} + \frac{\lambda^2_{it} - \mu_i}{(1-\gamma)\mathbb{E}_\omega\left[R^E_i\right]^{-\gamma}}$$

where

$$\mathbb{E}_\omega[\omega_{it} r^x_{it}] = \chi^L_{it} \int_{-\infty}^{\frac{c_{it}}{d_{it}}} \omega f(\omega)\, d\omega + \chi^B_{it} \int_{\frac{c_{it}}{d_{it}}}^{1} \omega f(\omega)\, d\omega$$

The first order conditions for an interior solution imply:

$$R^b_{it} = \frac{\mathbb{E}_\omega \left[r^x_{it} \cdot \left[R^E_i\right]^{-\gamma} \right]}{\mathbb{E}_\omega \left[R^E_i\right]^{-\gamma}} \tag{83}$$

$$R^b_{it} - R^d = \frac{\mathbb{E}_\omega \left[r^x_{it} \cdot \omega_{it} \cdot \left[R^E_i\right]^{-\gamma} \right] + \frac{\mu_i}{1-\gamma}}{\mathbb{E}_\omega \left[R^E_i\right]^{-\gamma}} \tag{84}$$

where μ_i is the multiplier on the capital requirement constraint.

$$R^b_{it} = \underbrace{\mathbb{E}_\omega r^x_{it}}_{\text{direct effect}} + \underbrace{\frac{\mathbb{COV}\left\{ r^x_{it}, \left[R^E_i\right]^{-\gamma} \right\}}{\mathbb{E}_\omega \left[R^E_i\right]^{-\gamma}}}_{\text{liquidity risk premium effect}}$$

$$R^b_{it} - R^d \geq \underbrace{\mathbb{E}_\omega \left[r^x_{it} \cdot \omega_{it} \right]}_{\text{direct effect}} + \underbrace{\frac{\mathbb{COV}\left\{ r^x_{it} \cdot \omega_{it}, \left[R^E_i\right]^{-\gamma} \right\}}{\mathbb{E}_\omega \left[R^E_i\right]^{-\gamma}}}_{\text{liquidity risk premium effect}}$$

where the latter holds with equality if the capital requirement is non-binding. The covariance terms are liquidity risk premia. For a risk-neutral bank ($\gamma = 0$) these terms disappear.

Once the optimal values for $\hat{c}_i$ and $\hat{d}_i$ are found, the expected value of $\left[R^E_i\right]^{1-\gamma}$ equals:

$$\Omega^*_{it} \equiv \mathbb{E}_\omega \left[R^E_i\right]^{1-\gamma} = F\left[\frac{\hat{c}^*_i}{\hat{d}^*_i}\right] \left[R^E_i\left(\chi^L_{it}\right)\right]^{1-\gamma} + \left(1 - F\left[\frac{\hat{c}^*_i}{\hat{d}^*_i}\right]\right) \left[R^E_i\left(\chi^B_{it}\right)\right]^{1-\gamma}$$

C.4 Dividends and Bank Value

The value function is linear in Ω^*_{it}:

$$v(Z) = \max_{div_i} \quad \frac{{div_i}^{1-\gamma}}{1-\gamma} + \beta\zeta\left(1 - div_i\right)^{1-\gamma} \mathbb{E}v\left(Z'|Z\right)\Omega^*_{it}$$

Differentiating w.r.t. div_i:

$$\begin{aligned}
{div_i}^{-\gamma} &= \beta\zeta(1-\gamma)\Omega^*_{it}\left(1 - div_i\right)^{-\gamma} \mathbb{E}\; v\left(Z'|Z\right) \\
\frac{1}{div_i} &= 1 + \left[\beta\zeta(1-\gamma)\mathbb{E}v\left(Z'|Z\right)\Omega^*_{it}\right]^{\frac{1}{\gamma}} \\
div_i &= \frac{1}{1 + \left[\beta\zeta(1-\gamma)\mathbb{E}v\left(Z'|Z\right)\Omega^*_{it}\right]^{\frac{1}{\gamma}}}
\end{aligned}$$

Substituting back to the value function results in the following functional equation:

$$v(Z) = \frac{1}{1-\gamma}\left[1 + \left[\beta\zeta\left(1-\gamma\right)\mathbb{E}v\left(Z'|Z\right)\Omega^*_{it}\right]^{\frac{1}{\gamma}}\right]^{\gamma} \tag{85}$$

The right-hand side can be treated as a contraction mapping operator. Once the value function is solved, next period equity can be calculated:

$$E'_{it} = \left(1 - div_i\right) E_{it} R^E_i$$

This concludes the bank problem.

D Real Sector

D.1 Household

Household obtains utility from consumption, C_t, and disutility from labor, H_t. The household can save by providing deposits to the banking sector, D^A_t. Deposits receive a constant interest rate of R^d, which is paid at the beginning of the next period.

Problem 7 *The household solves the following maximization problem:*

$$\max_{C_t, H_t, D_t^A} \quad \sum_{t=0}^{\infty} \beta^t \left[C_t - \frac{H_t^{1+\nu}}{1+\nu} \right]$$
$$s.t. \qquad D_t^A + C_t = W_t H_t + R^d D_{t-1}^A + \Pi_t + T_t$$

where W_t is the real wage rate, Π_t is the firm's profit, T_t is the tax transfer, and ν is the inverse of the Frisch elasticity. The labor supply curve is:

$$H_t = W_t^{\frac{1}{\nu}} \tag{86}$$

which implies that the household's total wage income is $W_t^{\frac{\nu+1}{\nu}}$. If $R^d = \frac{1}{\beta}$, the household is indifferent between consumption and saving, and:

$$C_t \in [0, Y_t], \quad D_t^A = Y_t - C_t, \quad R^d = \frac{1}{\beta} \tag{87}$$

where Y_t is firm's output.

D.2 Firm

An aggregate profit-maximizing firm uses household's labor to produce output according to the following production function:

$$Y_t = A_t H_t^{1-\alpha} \tag{88}$$

where A_t is a technology index, and $1-\alpha$ is the labor share. The firm has to pay workers before output is realized, therefore it borrows the total amount I_t^A from the banking sector to cover the wage bill:

$$W_t H_t = I_t^A \tag{89}$$

I_t^A is collected via the CES technology:

$$I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{90}$$

where I_{it} is borrowing from bank i, λ_i is the bank i's share, and ϵ is the elasticity of substitution between loans from different banks. The firm promises to repay the loan principal and accrued interest in the beginning of the next period. The total repayment to the banking sector is then $\sum_i R_{it}^b I_{it}$. The firm never defaults on loans.

Problem 8 *The aggregate firm solves the following maximization problem:*

$$\begin{aligned}
\max_{I_t^A, I_{it}, H_t} \quad & \sum_{t=0}^{\infty} \beta^t \left[A H_t^{1-\alpha} - W_t H_t + I_t^A - \sum_i R_{itit-1}^b I_{it-1} \right] \\
s.t. \quad & W_t H_t = I_t^A \\
& I_t^A = \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}$$

Substituting the constraints into the objective, the firm's problem can be written as an unconstrained maximization problem:

$$\max_{I_{it}} \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{A_t}{W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}} - \sum_i R_{itit-1}^b I_{it-1} \right]$$

The first-order condition implies the demand curve for a loan from a bank i:

$$\begin{aligned}
R^b_{it} &= \frac{(1-\alpha)\,A_t}{\beta W_t^{1-\alpha}}\left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}-1} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta W_t^{1-\alpha}}\left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon(1-\alpha)-\epsilon+1}{\epsilon-1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta W_t^{1-\alpha}}\left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1-\alpha\epsilon}{\epsilon-1}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta W_t^{1-\alpha}}\left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1-\epsilon\alpha}{\epsilon}} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}}
\end{aligned}$$

Equivalently,

$$R^b_{it} = \frac{(1-\alpha)\,A_t}{\beta W_t^{1-\alpha}}(I^A_t)^{\frac{1}{\epsilon}-\alpha}\lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \tag{91}$$

D.3 Labor Market Clearing

Substituting the labor supply condition (86) into the working-capital constraint (89) gives the relationship between wages and total investment:

$$W_t^{\frac{1+\nu}{\nu}} = I^A_t$$

Substituting the above in the loan demand (91):

$$\begin{aligned}
R^b_{it} &= \frac{(1-\alpha)\,A_t}{\beta W_t^{1-\alpha}}(I^A_t)^{\frac{1}{\epsilon}-\alpha}\lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta H^{\nu(1-\alpha)}}(I^A_t)^{\frac{1}{\epsilon}-\alpha}\lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta (I^A_t)^{\frac{\nu(1-\alpha)}{\nu+1}}}(I^A_t)^{\frac{1}{\epsilon}-\alpha}\lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)}{\beta}A_t\left[I^A_t\right]^{\frac{1}{\epsilon}-\alpha-\frac{\nu(1-\alpha)}{\nu+1}}\lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\left[\frac{I_{it}}{\lambda_i}\right]^{-\frac{1}{\epsilon}}
\end{aligned}$$

Simplifying:

$$R^b_{it} = \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\left[\frac{I_{it}}{\lambda_i}\right]^{-\frac{1}{\epsilon}} \tag{92}$$

$$\begin{aligned}
R^b_{it} &= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\lambda_i^{\frac{1}{\epsilon}}I_{it}^{-\frac{1}{\epsilon}} \\
R^b_{it}I_{it} &= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\lambda_i^{\frac{1}{\epsilon}}I_{it}^{-\frac{1}{\epsilon}}I_{it} \\
R^b_{it}I_{it} &= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\lambda_i^{\frac{1}{\epsilon}}I_{it}^{\frac{\epsilon-1}{\epsilon}}
\end{aligned}$$

Summing over i:

$$\begin{aligned}
\sum_i R^b_{it}I_{it} &= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\sum_i \lambda_i^{\frac{1}{\epsilon}}I_{it}^{\frac{\epsilon-1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\left[\left[\sum_i \lambda_i^{\frac{1}{\epsilon}}I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}\right]^{\frac{\epsilon-1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}}\left[I^A_t\right]^{\frac{\epsilon-1}{\epsilon}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1}{\epsilon}+\frac{\epsilon-1}{\epsilon}-\frac{\nu+\alpha}{\nu+1}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{1-\frac{\nu+\alpha}{\nu+1}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{\nu+1-\nu-\alpha}{\nu+1}} \\
&= \frac{(1-\alpha)\,A_t}{\beta}\left[I^A_t\right]^{\frac{1-\alpha}{\nu+1}} \\
&= \frac{1-\alpha}{\beta}Y_t
\end{aligned}$$

The firm's profit is then:

$$\Pi_t = \frac{A_t}{W_t^{1-\alpha}} \left[\sum_i \lambda_i^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}} - \sum_i R^b_{itit-1} I_{it-1}$$

$$\Pi_t = \frac{A_t}{W_t^{1-\alpha}} \left[I_t^A \right]^{1-\alpha} - \frac{(1-\alpha) A_{t-1}}{\beta} \left[I_{t-1}^A \right]^{\frac{1-\alpha}{\nu+1}}$$

$$\Pi_t = A_t H_t^{1-\alpha} - \frac{(1-\alpha) A_{t-1}}{\beta} \left[I_{t-1}^A \right]^{\frac{1-\alpha}{\nu+1}}$$

$$\Pi_t = A_t [I_t^A]^{\frac{1-\alpha}{1+\nu}} - \frac{(1-\alpha) A_{t-1}}{\beta} \left[I_{t-1}^A \right]^{\frac{1-\alpha}{\nu+1}}$$

$$\Pi_t = Y_t - \frac{1-\alpha}{\beta} Y_{t-1}$$

E Equilibrium Conditions

$$(1-\theta)\, E_{it} = \tilde{B}_{it} + \tilde{C}_{it} - \tilde{D}_{it}$$

$$\tilde{D}_{it} \leq \kappa\,(1-\theta)\, E_{it}$$

$$E_{it+1} = R^b_{it}\tilde{B}_{it} - R^d \tilde{D}_{it} + R^x_{it}\tilde{L}_{it} - \omega_{it}\left(R^x_{it} - R^d\right)\tilde{D}_{it}$$

$$X_{it} = \omega_{it}\tilde{D}_{it} - \tilde{L}_{it}$$

$$R^x_{it} = \begin{cases} \chi^L_{it} = p^L_{it} r^{FF}_t + \left(1-p^L_{it}\right) r^{ER}_t & \text{if} \quad X_{it} \leq 0 \\ \chi^B_{it} = p^B_{it} r^{FF}_t + \left(1-p^B_{it}\right) r^{DW}_t & \text{if} \quad X_{it} > 0 \end{cases}$$

$$R^b_{it} = \chi^L_{it}\, F\left[L_{it}\right] + \chi^B_{it}\left(1 - F\left[L_{it}\right]\right)$$

$$R^b_{it} - R^d = \chi^L_{it}\int_{-\infty}^{L_{it}} \omega_{it} f\left(\omega_{it}\right) d\omega_{it} + \chi^B_{it}\int_{L_{it}}^{1} \omega_{it} f\left(\omega_{it}\right) d\omega_{it} + \mu_{it}$$

$$D^A_t + C_t = W_t H_t + R^d D^A_{t-1} + \Pi_t + T_t$$

$$H_t = W_t^{\frac{1}{\nu}}$$

$$D^A_t = Y_t - C_t \text{ and } R^d = \frac{1}{\beta}$$

$$Y_t = A_t H_t^{1-\alpha}$$

$$I^A_t = B_{it} = \left[\sum_i \lambda^{\frac{1}{\epsilon}} I_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$W_t H_t = I^A_t$$

$$R^b_{it} = \frac{(1-\alpha)\, A_t}{\beta W_t^{1-\alpha}} (I^A_t)^{\frac{1}{\epsilon}-\alpha} \lambda_i^{\frac{1}{\epsilon}} I_{it}^{-\frac{1}{\epsilon}}$$

$$\Pi_t = A_t H_t^{1-\alpha} - \sum_i R^b_{itt-1} I_{it-1}$$

$$M^0_{t+1} - M^0_t = D^{CB}_{t+1} - D^{CB}_t - r^{DW}_t X^-_t + r^{ER}_t X^+_t + T_t$$

$$X^-_t = \sum_i \mathbb{1}_{\{X_{it}>0\}} \cdot \left(1 - p^B_{it}\right) X_{it}, \quad X^+_t = \sum_i \left(1 - \mathbb{1}_{\{X_{it}>0\}}\right) \cdot \left(1 - p^L_{it}\right) X_{it}$$

$$D^A_t = \sum_i \tilde{D}_{it}$$

$$\sum_i \tilde{C}_{it} = D^{CB}_t = M^0_t$$